THE DEMAND FOR DURABLES, NONDURABLES, SERVICES AND THE SUPPLY OF LABOUR IN CANADA: 1946 - 1969

by
Thomas Keith Gussman

University of British Columbia
and
Research Branch
Program Development Service
DEPARTMENT OF MANPOWER AND IMMIGRATION
CANADA
1972

THE DEMAND FOR DURABLES NON-DURABLES, SERVICES AND THE SUPPLY OF LABOUR IN CANADA

by

Thomas K. Gussman

Research Branch,
Department of Manpower and
Immigration.

Department of Economics,
University of British
Columbia.

November 1972.

OTHER PAPERS IN THIS SERIES

1. DIEWERT, W.E. - Functional Forms for Revenue and Factor Requirements Functions.

2. DIEWERT, W.E. - The Effects of Unionization on Wages and Employment: A General Equilibrium Analysis.

3. DIEWERT, W.E. - Walras' Theory of Capital Formation and the Existence of a Temporary Equilibrium.

4. DIEWERT, W.E. - Functional Forms for Profit and Transformation Functions.

5. AHAMAD, B. & SCOTT, K.F.N. - A Note on Sensitivity Analysis in Manpower Forecasting.

6. EPSTEIN, L. - Report on a Demographic Model to Project Population by Age, Sex, Education and Province.

7. DIEWERT, W.E. - Choice on Labour Markets and the Theory of the Allocation of Time.

8. EPSTEIN, L. - Some Economic Effects of Immigration: A General Equilibrium Analysis.

9. WOODLAND, A.D. - Factor Demand Functions for Canadian Industries 1946 - 1969.

10. GUSSMAN, Thomas Keith - Demand for Durables, Nondurables, Services and the Supply of Labour in Canada: 1946 - 1969.

11. DIEWERT, W.E. - Canadian Labour Markets: A Neoclassical Economic Approach.

TABLE OF CONTENTS

LIST OF TABLES

A C K N O W L E D G E M E N T

It would be grossly unfair to proceed without first paying tribute to a long list of people who have unselfishly offered ideas and criticisms throughout the formative stages of this thesis. Among these, Professors Robert Evans, John Helliwell, Ronald Shearer, William Stanbury, and Dan Usher were constantly plagued by my queries and deserve a note of thanks for their suggestions and patience. My committee, consisting of my supervisor, Professor Erwin Diewert, and Professors Terence Wales and Alan Woodland, somehow managed to endure the trial-and-error process that is inherent in the data game, and hopefully, they have emerged relatively unscarred. I owe them an especial debt of gratitude. Professor Diewert's theories and teachings are self-evident in my work, and without him I would quite likely have been unable to work on the present topic, namely, the simultaneous estimation of consumer demand and labour supply functions.

It is important to emphasize that all persons mentioned should not be associated with any errors, since I alone am responsible.

The typing of the manuscript (especially the interpretation of my handwriting!) was no small task, and Susan Aizenman, whose "cliff hanger" style has erupted many an ulcer, came through in fine style with eleven minutes to spare before the deadline. In typing the final draft, my wife, Natalie, provided an inspiration to us by incessantly talking so as to prevent our sleeping on the job.

Finally, I wish to acknowledge the Department of Manpower and Immigration (Ottawa) for financial support.

CHAPTER I

INTRODUCTION

In the literature of applied consumer theory, several attempts have been made to approximate the parameters of "the" utility function underlying the expenditure patterns observed in a particular country. Most notably, the works of Stone (Great Britain); Houthakker and Taylor (U.S.A.); Pollak and Wales (U.S.A.); Schweitzer (Canada), and Wales (Canada) have been attempts to estimate the preference map underlying demands emergent from an inherent maximization of some particular function subject to an income or budget constraint. At times, the demand functions have been linear; semi-logarithmic; double-logarithmic; or even inverse semi-logarithmic. Some of these functions have constant elasticities of substitution, while others admit the possibility of an arbitrary elasticity of substitution between each pair of goods. Whatever the functional form, these studies have each contributed to the literature. In a similar fashion, the present study attempts to estimate the "typical" Canadian consumer's utility map in a formalized revealed preference framework. We make use of the indirect utility function and assume a non-homothetic Generalized Leontief functional form due to Diewert [7; 1971] which is quadratic in the square roots of the parameters (in the consumer case prices). From the derived demand equations, we estimate the preferences in a generalized least squares procedure.

Our model has several pitfalls, among which is the problem of dealing with aggregate data and arbitrarily dividing all quantities by population to approximate per-capita shares. Clearly, whether we *evenly* distribute capital stock and expenditure by person or by family, we are not obtaining a true picture of personal expenditure. As is well-known, statistical difficulties may arise in such a framework. However, since a survey of *all* families in the data universe is infeasible, we work with what is available.

Another shortcoming in this study is the exclusion of personal savings from the model; here, savings is an exogenous decision and is thus not determined as a decision variable along with expenditures and labour supply.

The good features, however, ought to, at least in part, compensate for the omissions. We are hopeful that the inclusion of a more detailed allocation of expenditure in the current period, the treatment of consumer durables in a user-cost framework, and the simultaneous estimation of the demand for "dis-work" (leisure) so that the labour supply decision is endogenous to the system, will prove to be useful.

Chapter II briefly reviews other studies in consumer theory. Chapter III outlines the Generalized Leontief Function, its properties and the process used to estimate its unknown parameters.

CHAPTER II

A REVIEW OF THE LITERATURE

A. THE LINEAR EXPENDITURE SYSTEM (LES)

From the Stone-Geary utility function,[1] we may derive demand equations in which expenditure on each good is a linear homogeneous function of income and all prices. For this reason the model is known as (Stone's) linear expenditure system. A typical Stone-Geary utility function is:

$$U = \sum_{i=1}^{n} \beta_i \log(X_i - \gamma_i) \qquad \text{(II-1)}$$

where

X_i is a quantity;

$0 \leq \beta_i < 1$;

for all i

$\sum_{i=1}^{n} \beta_i = 1.$

The function is only defined when $(X_i - \gamma_i) > 0$. This function is directly additive,[2] and the resultant demand system is:

$$p_i X_i = p_i \gamma_i + \beta_i \cdot \left(y - \sum_{j=1}^{N} p_j \gamma_j\right) \quad (i=1,\ldots,N) \qquad \text{(II-2)}$$

---

[1] The points brought out here are basically condensed from Goldberger's [18] superlative survey of consumer demand theory (i.e., re the Stone-Geary function).

[2] Goldberger [18; p. 45] shows this algebraically.

where

y = income;

p_i = price of ith good.

Goldberger offers a useful interpretation:

> Given income y and prices $p_1, . . ., p_n$, the consumer first purchases "minimum required quantities" of each good: γ_1 of good 1,. . ., γ_n of good n. At the given prices, this costs $\sum_j p_j \gamma_j$, which may be termed "subsistence income." He is left with $y - \sum_j p_j \gamma_j$, which may be termed "supernumerary income"; this he distributes among the goods in the proportions $\beta_1, ... \beta_n$. (The β's are thus identified with. . .marginal budget shares...) Recall that the utility function is defined only for $(q_i - \gamma_i) > 0$ $(i=1,. . ., n)$; it makes no prediction about the behaviour of a consumer for whom $y - p_j \gamma_j = \sum_j p_j (q_j - \gamma_j) < 0$ since he cannot afford to acquire $\gamma_1, . . ., \gamma_n$ at the going prices.[3]

Of course, there is no need to interpret the γ as bare minima, but this particular form of utility function only requires that $(x_i - \gamma_i) > 0_i$ and $y - \sum_j p_j \gamma_j > 0$.

One further point to note is that there can be no inferior goods in the LES. Stone, either by himself or in conjunction with others, has estimated this LES extensively using British data.

B. THE GENERALIZED LINEAR EXPENDITURE SYSTEM

Wales [37], following work done jointly with Pollak (in the United States), estimates a *generalized* linear expenditure model of the demand for nondurables in Canada. The utility function is assumed to be of

---

[3]Goldberger [18; p. 47].

the form:

$$U(x) = \left(\sum_{i=1}^{n} a_i (x_i - b_i)^{\alpha}\right)^{1/\alpha} \qquad \text{(II-3)}$$

where

$a_i > 0;$

$x_i - b_i \geq 0;$

$\alpha < 1;$

$x_i \geq 0.$

The utility-maximizing process yields demand functions of the form:

$$x_i = b_i + [(a_i/p_i)^c / \sum_{j=1}^{n} (a_j/p_j)^c p_j] * (y - \sum_{j=1}^{n} p_j b_j) \qquad \text{(II-4)}$$

where

$a_i > 0;$

$x_i - b_i > 0$

$x_i > 0$

c = elasticity of substitution.

As Wales points out, the system differs from the LES basically in that (i) marginal budget shares are price dependent rather than constant; and (ii) partial elasticities of substitution[4] between any pair of supernumerary quantities $(x_i - b_i)$ which equal 1 in the LES, are equal to c in the Wales system.

---

[4] In the present study, these elasticites are denoted as σ_{ij}.

For estimation purposes, a "habit formation" concept is adopted as a dynamic specification; this is achieved by allowing the b's to depend on previous consumption.[5]

The empirical work is carried out under various stochastic specifications on the broad consumption aggregates of food, clothing, shelter and miscellaneous. The results deem a proportional habit model inconsistent with the underlying utility function, but are consistent with a linear (lagged) habit model. In his conclusions, Wales suggests that while the added feature of the estimation of c (the elasticity of substitution between pairs of supernumerary quantities) in the generalized LES *does* make the system more flexible, the results are not significantly different enough to warrant using the generalized LES rather than the LES when a large number of goods or observations are involved, given that the generalized model carries with it computational difficulties.

C. THE DEMAND FOR DURABLE GOODS

Since one of the interesting features of the current study is a reasonably thorough treatment of consumer durables, it will prove useful to examine other approaches to the demand for durables.

Stone and Rowe [35; 1957] first proposed a simple, testable dynamic theory of demand (for durables). The demand for "perishables" (nondurable

---

[5] Wales [37; p. 473]: "This means that the utility functions are no longer separable over time and that the demand equations should be obtained assuming intertemporal utility maximization. We avoid this complication by assuming that the consumer is unaware that his future preferences change as a function of his current behaviour."

goods and services) is a special case in which one of the parameters (obviously the rate of depreciation) goes to its limit. Basically, gross expenditure on a durable good may be decomposed into an addition to the opening stock (i.e., an investment) and a flow of consumption (i.e., what is used up during the period). The authors assume a reducing-balance depreciation formula and thus the amount used up (u) will be 1/nth of the opening stock(s).

Accordingly,

(i) $Q = v + u$

(ii) $u = \frac{s}{n} + \frac{Q}{m}$ (Q represents total purchases or gross investment and v net investment) (II-5)

$$\equiv \frac{m}{n(m-1)} s + \frac{1}{m-1} v$$

Here, $\frac{1}{m}$ is that proportion of the current period purchases Q which is used up.

Generally, it is assumed that $m > n$ and $n > 1$. The closing stock (Es) equals the opening stock (s) plus net investment (v) ($Es \equiv s+v$).

"The crucial feature of the theory is a distinction between the opening stock, s, and the equilibrium stock, s*, which is assumed to depend on consumers' incomes and prices."[6] It is assumed that net investment is undertaken to reduce the gap (between s and s*) by a proportion $\frac{mr}{n}$. Thus

$$v = \frac{mr}{n} (s^*-s) \qquad \text{(II-6)}$$

where r is taken to be the rate of stock adjustment.

---

[6] Stone and Rowe [35; p. 425].

This method of stock-adjustment has been popular in studies conducted throughout the 1960's. The authors give the best summary in their own words:

> . . .The theory employs a reducing balance depreciation formula with a depreciation rate that is constant over time, a concept of equilibrium consumption which depends, apart from a residual trend, on current income and the current price structure and an investment relationship designed to narrow and, under stable conditions, eventually to eliminate the difference between equilibrum and actual stocks.[7]

A distinguished group of Chicago economists[8] formed in 1954 conducted a series of studies on the cyclical process in the United States. In particular, these studies focus on the durables sector since the wide fluctuations in durables demand make that sector a "danger spot" (in which wide cyclical swings could initiate). Thus, as Harberger puts it, "an understanding of the mechanisms determining changes in the demand for durable goods is important for estimating the probable effects of proposed stabilization schemes."[9]

These studies, ranging from refrigerators to non-farm housing, have been collected in [21a] and edited by Harberger. One of the main features is the treatment of the stock demand versus flow demand problem. There remains, however, the concept of a "desired" stock to which consumers will supposedly adjust. Many problems are pointed out and attempts are made to

---

[7] Stone and Rowe [35; p. 441].

[8] A. C. Harberger, R. F. Muth, M. C. Burstein, G. C. Chow, Z. Griliches, and Y. Grunfeld. See reference [21a].

[9] Harberger (Ed.), [21a], p. v.

treat such problem areas as thoroughly as possible given the constraint of imprecise data (relative to the reasonably exact data available, for example, in the area of wheat consumption). As Harberger points out:

> . . .If cars of all ages really have the same service yield, the aggregate service yield of the existing stock of cars would be measured by their number; if, on the other hand, the service yield of individual cars is proportional to their value, the service yield of the stock would be measured by its aggregate value.[10]

Thus, if automobile depreciation were a constant dollar amount each year, this would be an indication that the flow of services was approximately the same for cars of different vintages, and the "numbers" measure would be more suitable. If, however, (which is the more likely case) depreciation were a constant percentage of the value of the stock, the "aggregate value" measure would be preferable. Even at that, either measure is only a rough approximation, since maintenance costs have the ability to raise the capacity of the existing stock. Thus, at best, we can only get an approximation to "the" stock of a particular durable. The overall contribution of this collection of studies is a careful sifting of the evidence in order to obtain answers to certain broad questions in the area of durables demand.

More recently, Houthakker and Taylor [25] and Schweitzer [33] have undertaken highly disaggegated econometric consumer demand studies in the United States and Canada respectively. The basic dynamic model is again the (equilibrium) stock-adjustment concept and depreciation of durables is assumed to be of the declining-balance form. The demand equations for non-

---

[10] Harberger (Ed.), [21a], p. 5.

durables are of the same form as those for durables with the concept of stock adjustment replaced by that of habit formation. As pointed out by Houthakker, the stock may be psychological instead of, or as well as, physical. Total personal consumer expenditure is taken to be the budget constraint, and a set of demand equations consistent with utility maximization is estimated. In the end, these demand equations are used to make projections.

In the U.S. study [25], prices are found to play a modest role relative to income in determining consumption. "If income is high enough, it is possible for nearly all commodities to become subject to habit formation."[11] The basic conclusion is that the dynamic model is, for the most part, here to stay.

D. THE INDIRECT UTILITY FUNCTION

The traditional studies have one common base; all of them, having assumed an arbitrary form for the utility function, maximize this utility function subject to a budget constraint and use the algebraic solution for derived demand equations as a basis for estimation. "The problem with this approach is that if one assumes a 'flexible'[12] functional form for the utility function, the derived demand functions are either impossible to derive algebraically or are extremely non-linear in the unknown parameters."[13]

---

[11] [25; p. 305].

[12] i.e., capable of providing a second order approximation to an arbitrary twice differentiable utility function. The corresponding derived demand functions will be approximated to the first order. (Diewert [9; p. 47]).

[13] *Ibid.*, pp. 19-20.

Under certain assumptions and regularity conditions, a much easier method for obtaining a system of derived demand functions is available -- namely, the indirect utility function approach.[14] Whereas the direct utility function $U=U(x)$ is difficult to estimate, the indirect utility function [which shows the maximum utility the consumer can achieve given "income" (Y) and prices (p)], can be readily estimated if we impose a normalization. In this study, we will work with prices *normalized* by income; we shall call these normalized prices $v_i \equiv p_i/Y$. Thus

$$g(v) \equiv \max_{\substack{\text{w.r.t.} \\ x \geq \underline{0}}} \{U(x): \sum_{i=1}^{n} v_i x_i \leq 1\} \qquad \text{(II-7)}$$

where

g = indirect utility function;

x = consumer's choice variables (quantities);

$\underline{0}$ = a zero vector;

$\Sigma v_i x_i$ = budget constrant.

Diewert [9; p. 20], following the work of Shephard[15] and Hanoch[16] demonstrates that from the duality between direct and indirect utility functions we may in fact calculate *the* utility function $U(x)$ as the solution to

---

[14] Diewert [7]; [9]; [10]; [12].

[15] R. W. Shephard, *Theory of Cost and Production Functions* (Princeton, N.J.: Princeton University Press, 1970) treats the indirect utility function in the appendix.

[16] Giora Hanoch, "Generation of New Production Functions Through Duality," mimeo (1970).

the following problem:

$$U(x) = \min_{\substack{\text{w.r.t.} \\ v \geq 0}} \{g(v): \sum_{i=1}^{n} v_i x_i \leq 1\} \tag{II-8}$$

The brief outline which follows is essentially condensed from Hanoch [21aa], Shephard [33b], and Diewert [9; p. 21].

"If the direct utility function U is non-negative, continuous, quasi-concave, and non-decreasing in the components of X, then the inverse indirect utility function, $h(v) \equiv 1/g(v)$, will have the same properties as the utility function and vice versa."[17] Non-negativity is self-explanatory. The non-decreasing requirement means that if $x^1 \geq x^2$, then $f(x^1) \geq f(x^2)$. Quasiconcavity (a weaker form of concavity) is obtained when the level sets $L(x) = \{x: f(x) \geq y \; \forall \; x \geq 0\}$ are convex over the non-negative orthant.

If, finally, $h(v)$ (the inverse indirect utility function) is once differentiable with respect to the prices v_i, then the consumer's system of derived demand equations becomes (provided the partial derivatives are not identically zero):

$$x_i(v) = \frac{\partial h(v)}{\partial v_i} / \sum_{k=1}^{n} v_k \frac{\partial h}{\partial v_k}(v); \text{ for all } i. \tag{II-9}$$

---

[17] [9; p. 21].

If all the necessary conditions are satisfied, we have a reasonably compact way of solving for derived demand equations consistent with the consumer's utility maximization.

One further step is necessary before we may assume a functional form. Consider the following utility maximization problem (where we assume the allocation of time problem has been solved):

$$U(X,L) = F(X,L)$$

$$\text{MAX } F(X_1, \ldots, X_N; L)$$

w.r.t.

$$X_1 \geq 0, \ldots, X_N \geq 0$$

$$L \geq 0 \qquad \text{(II-10)}$$

subject to

$$\text{(i)} \quad \sum_{i=1}^{n} p_i X_i - wL \leq V$$

$$\text{(ii)} \quad L \leq H$$

where

X_i = number of units of the services of the ith good the consumer-worker purchases during the period;

p_i = the price (or rental price for durable goods) of one unit of good i;

w = the market wage rate for our one type of labour service the consumer-worker is qualified to supply;

L = the number of hours of labour the consumer-worker supplies during the period;

V = non-labour income spent during the period;

H = total number of *available* hours in the period.

In (II-10) the consumer has preferences defined over the services of consumption goods and labour supply. However, the method for obtaining derived demand functions will fail since we do not meet all of the following requirements:

(i) $U = f(X;L)$ must be defined over the entire non-negative orthant;

(ii) all prices must be non-negative (the wage rate enters negatively);

(iii) full "income" must be positive.

A simple transformation of our variables will achieve the desired properties. We need only redefine the utility function over the demand for "diswork" (leisure) rather than the *supply* of labour services, and our new utility function is acceptable:

$$\begin{aligned}
&U(X,H-L) \equiv f(X_1,\ldots,X_N;L)\\
&\text{MAX } U(X,H-L)\\
&\text{w.r.t.}\\
&X_1 \geq 0,\ldots,X_N \geq 0\\
&H-L \geq 0
\end{aligned} \tag{II-11}$$

subject to:

$$\text{(i)} \quad \sum_{i=1}^{n} p_i X_i + w(H-L) \leq V + wH$$

where there are n consumer goods and only one type of labour service;

$$\text{(ii)} \quad (H-L) \leq H$$

Assuming the time constraint is satisfied, the problem becomes:

$$\text{MAX } U(X)$$

$$\text{w.r.t. } X \geq \underline{0} \qquad \text{(II-}$$

subject to

$$\sum_i v_i X_i \leq 1$$

where

$$X = (X_1, \ldots, X_N; H-L)$$

$$v = \frac{1}{V + wH} (p_1, \ldots, p_n; w)$$

For estimation purposes, the "generalized full income" variable V + wH is taken to be the value of the left-hand side of [II-11(i)]; that is, we assume the budget constraint is fully satisfied.

The final step before empirical work is to assume a functional form for h(v), the inverse indirect utility function.

CHAPTER III

GENERALIZED LEONTIEF PREFERENCES AND THEIR ESTIMATION

A. A GENERALIZED LEONTIEF UTILITY FUNCTION

The Generalized Leontief functional form (due to Diewert [7]) first arose in the context of production theory. This particular function first appeared in the homothetic[1] case, but we will work with a non-homothetic version.

Consider the following form for the inverse indirect utility function $h(v)$:

$$h(v_1, v_2, \ldots, v_{N+K})$$

where there are N consumer goods and K types of labour service

$$= \sum_{i=1}^{N+K} \sum_{j=1}^{N+K} b_{ij} v_i^{\frac{1}{2}} v_j^{\frac{1}{2}} + 2 \sum_{j=1}^{N+K} b_{oj} v_j^{\frac{1}{2}} \qquad \text{(III-1)}$$

where

$$b_{ij} = b_{ji} \ \forall \ i \neq j$$

Remembering that the consumer's preferences extend over goods *and* labour services, we may generate a system of derived demand equations (where $x_{N+K}(v) \equiv H-L_K$ is a typical labour supply function):

---

[1]A utility function $u(x)$ is homothetic if there exists an increasing function of one real variable g such that $g(u(x))$ is homogeneous of degree one. (Diewert [9; p. 98]; originally defined by Shephard 1953 [33a]).

$$x_i(v) = \frac{\{\sum_{j=1}^{N+K} b_{ij} v_j^{\frac{1}{2}} v_i^{-\frac{1}{2}} + b_{oi} v_i^{-\frac{1}{2}}\}}{\{\sum_{k=1}^{N+K} \sum_{j=1}^{N+K} b_{kj} v_k^{\frac{1}{2}} v_j^{\frac{1}{2}} + \sum_{j=1}^{N+K} b_{oj} v_j^{\frac{1}{2}}\}} \qquad \text{(III-2)}$$

V_i

The derived demand functions (III-2) are homogeneous of degree zero in the unknown parameters b_{ij} and we are allowed one normalization:

$$\sum_{i=1}^{N+K} \sum_{j=1}^{N+K} b_{ij} + \frac{2}{k} \sum_{j=1}^{N+K} b_{oj} = 1 \qquad \text{(III-3)}$$

where $k = v^{*\frac{1}{2}}$; $v^* = v_i V_i$ in 1961. This was done to ensure that g (indirect utility) would be positive in 1961. In fact, our normalization sets $g = \frac{1}{v^*}$ in 1961.

"If $b_{oj} = 0$ for $j = 1,2,\ldots,N+K$, then the [corresponding] direct utility function. . .reduces to the (homothetic) generalized Leontief utility function; if in addition, $b_{ij} = 0$ for all $i \neq j$, then the direct utility function reduces to the ordinary fixed proportions Leontief utility function." [9; p. 23].

The derived demand equations (III-2) may be equivalently represented (for estimation purposes) by performing the following operations:

(i) multiply both sides of equation i (i=2, . . .,N+K) by

$$\left(\sum_{k=1}^{N+K} \sum_{j=1}^{N+K} b_{kj} v_k^{\frac{1}{2}} v_j^{\frac{1}{2}} + \sum_{j=1}^{N+K} b_{oj} v_j^{\frac{1}{2}}\right) \frac{v_i}{v_1}$$

(ii) use relation (III-3) (i.e., normalization) to solve for b_{11} in terms of the other b_{ij} and use this result to eliminate b_{11} from the N+K-1 transformed equations;

(iii) impose the symmetry conditions $b_{ij} = b_{ji}$ and collect all terms in the (unknown) b_{ij} on the right hand side of equations 2, . . .,N+K.

We thus arrive at a system of consumer demand equations (III-4) which expresses 'market' shares (i.e., the share of total expenditure) as a function of the unknown parameters and the normalized prices.

$$
\begin{aligned}
& v_i x_i \quad (i=2,\ldots,N+K) \\
&= \sum_{n=1}^{N+K} b_{on}\left\{(v_i x_i - v_n^{\frac{1}{2}} v_i x_i / v_1) + \delta_{in} \frac{v_i^{\frac{1}{2}}}{v_1}\right\} \\
&+ \sum_{1 \le k < j \le N+K}\sum b_{kj}\,(2v_i x_i - 2v_k^{\frac{1}{2}} v_j^{\frac{1}{2}} v_i x_i / v_1 \\
&\qquad + \delta_{ik}\, v_i^{\frac{1}{2}} v_j^{\frac{1}{2}} / v_1 \\
&\qquad + \delta_{ij}\, v_k^{\frac{1}{2}} v_i^{\frac{1}{2}} / v_1) \\
&+ \sum_{\ell=2}^{N+K} b_{\ell\ell}\,(v_i x_i - v_\ell\, v_i x_i / v_1 + \delta_{i\ell} \frac{v_i}{v_1}) \qquad \text{(III-4)}
\end{aligned}
$$

where

$$\delta_{ij} = \begin{cases} 0 \text{ if } i \neq j \\ 1 \text{ if } i = j \end{cases}$$

We may use linear regression techniques to estimate the unknown b_{ij}'s in (III-4) "if we assume that the distribution of the expenditure shares $v_i x_i$ conditional on the values of the right hand side variables, is multivariate normal."[2]

---

[2]Diewert [9; p. 25].

The inverse indirect utility function (III-1) has the same first and second order partial derivatives (evaluated at any particular point) as the indirect utility function; these derivatives are:

NOTATION: let h_i represent $\frac{\partial h(\bar{v})}{\partial v_i}$

and h_{ij} represent $\frac{\partial^2 h(\bar{v})}{\partial \bar{v}_i \, \bar{v}_j}$

$$h_i = \sum_{j=1}^{N+K} b_{ij}(\bar{v}_j/\bar{v}_i)^{\frac{1}{2}} + b_{oi}\,\bar{v}_i^{-\frac{1}{2}}$$

$$h_{ij} = \tfrac{1}{2} b_{ij}(\bar{v}_i\bar{v}_j)^{-\frac{1}{2}}$$

$$h_{ii} = -\tfrac{1}{2}\sum_{\substack{j=1\\ j\neq i}}^{N+K} b_{ij}(\bar{v}_i)^{-3/2}(\bar{v}_j)^{\frac{1}{2}} - \tfrac{1}{2}\, b_{oi}(\bar{v}_i)^{-3/2} \qquad \text{(III-5)}$$

We may use these partial derivatives to evaluate pseudo elasticities of substitution between the unnormalized demands i and j:
[The unnormalized demands are defined as $h_i(v)$].

$$\hat{\sigma}_{ij} \equiv h(v)h_{ij}(v)/h_i h_j \qquad \text{(III-6)}$$

Here, $\hat{\sigma}_{ij} = 0$ iff $b_{ij} = 0$ $\quad i \neq j$

If we have *a priori* beliefs that the unnormalized demand for good i is not responsive to changes in the price v_j, we may set $b_{ij} = 0$. This will add degrees of freedom to our model and reduce multicollinearity. A 'normalized' demand system is obtained by dividing each demand function by

$$\sum_{\ell=1}^{N+K} v_\ell h_\ell = \text{market demand.}$$

The formula for σ_{ij} (Hicks-Allen elasticities of substitution) for (market) demands is:

$$\sigma_{ij} = \frac{\sum_{k=1}^{N+K} v_k h_k(v) h_{ij}(v)}{h_i(v) h_j(v)} - \sum_{k=1}^{N+K} \frac{v_k h_{jk}(v)}{h_j(v)}$$

$$- \sum_{k=1}^{N+K} \frac{v_k h_{ik}(v)}{h_i(v)} + \sum_{m=1}^{N+K} \sum_{k=1}^{N+K} \frac{v_k h_{km}(v) v_m}{\sum_{k=1} v_k h_k(v)} \qquad \text{(III-7)}$$

As Diewert points out, even if consumers do *not* optimize according to our model (see Chapter II), "we can still assert that there are several advantages to fitting [this] system of consumer demand and labour supply functions. . .For example, the demand functions (III-2) are homogeneous of degree zero in prices $(p_1,. . .,p_N)$, wages $(w_1,. . .,w_K)$, and nonlabour income V. Also the consumption demands and labour supplies. . .are consistent with the consumer's budget constraint. Finally, the system of derived demand equations. . .can provide a good local approximation to an arbitrary system of derived demand functions, consistent with utility maximizing behaviour."[3]

B. ECONOMETRICS IN BRIEF

The unknown parameters were estimated by means of a two-stage procedure due to Zellner [41].

---

[3] Diewert[9; p. 27].

Our model is of the form:

$$y_t = X_t b + \varepsilon_t; \; t=1,. . .,T \tag{III-8}$$

where

y_t is a row vector of expenditure shares in period t;

X_t is a row vector of the values of the independent variables (calculated on the right-hand side of (III-4);

b is the matrix of unknown coefficients; and

ε is a row vector of stochastic disturbance terms.

Our stochastic specification is:

(i) $E(\varepsilon_t) = 0$; ε_t is normally distributed

(ii) $E(\varepsilon_t' \varepsilon_t) = \Sigma$

(iii) $E(\varepsilon_t' \varepsilon_s) = 0$

where ε_t' signifies the transpose of ε_t.

Σ is assumed to be a *singular* variance-covariance matrix since we are restricted to always be on the budget constraint. We also assume zero intertemporal covariances. We then "stack" the demand equations so that we have a system with (N+K-1)T observations. Thus, we have dropped one equation, and we run an ordinary least squares regression on the 'stacked' system and calculate the residuals.

From the residuals, we may obtain an *estimate* of E $(\hat{\Sigma})$; we use $\hat{\Sigma}$ to transform[4] the data; and we solve a generalized least squares problem with the transformed data. The GLS estimates of the coefficients of our consumer demand system are, supposedly, more efficient than would be in the single-

---

[4]The reader is referred to A. S. Goldberger, *Econometric Theory* (Wiley, 1964) p. 234, for a concise description of the transformation process.

equation estimation case. This is supported by the t-statistics, always higher for the coefficients of the "second stage" regression than for those of the first stage. The coefficients reported in Chapters V and VI are those from the (2nd stage) generalized least squares models. For a detailed description of the process and its asymptotic properties, see Zellner [41; pp. 349-352].

A comment on our stochastic specification is in order:

We may justify the specification of zero intertemporal covariances by assuming the existence of a separable intertemporal utility function.

In Chapter IV, we discuss the data manipulations (and sources) necessary for estimation of our consumer demand system.

CHAPTER IV

DATA: CONCEPTS AND SOURCES

A. THE TREATMENT OF DURABLE GOODS

Since we must decompose gross expenditure on a durable good into an investment component and a flow of services (rental) component, we must have data (insofar as durables are concerned) for the following:

1. The *real* stock of each type of durable (XD_i);

2. Current dollar gross investment expenditure on each durable good $\$DE_i$ (this should include maintenance expenditure since this presumably replenishes part of the existing stock);

3. The one-period depreciation rate (δ_i) assumed to hold over *all vintages* of each durable good, i.e., we assume a perpetual inventory model of capital stock;

   This means that if we are left with 4% of the stock of durable i from eight years ago, the units are indistinguishable from current purchases of the good; we only know that some units have deteriorated away;

4. A stock-price deflator PD_i^* to bring (2) into constant-dollar terms; or, in place of (2) and (4) a constant dollar series;

5. A series of "interest" rates R by which to discount future expected prices. The R will be opportunity costs.

1. Stocks

The (real) stock of durable i (XD_i) is defined as:

$$XD_{i_t} = XD_{i_{t-1}}(1-\delta_i) + \$DE_i/\overset{*}{PD}_i;$$

which is the balance of last period's carried-over stock (assumed to depreciate instantaneously in the middle of the consumer's time period) plus real new investment in the durable good (also assumed to be made in the middle of the period). This undepreciated balance carries over until the middle of next period, at which instant the process repeats itself. This is slightly different from the Stone and Rowe [35] method used in Helliwell *et al* [23] and Hamburger [21], which depreciates new investment by one-half of the per period rate immediately upon purchase. The program 'DELTA' (see Appendix) calculates the stock from a (constant-dollar) series of expenditure data, an initial stock (benchmark), and a depreciation rate.

2. Initial Stocks

Save for the cases of housing and land, where the initial stocks are based on early surveys and estimates, the initial stocks for our base year (1946) have been calculated in the following manner:[1]

> A reasonable approximation to "the" depreciation rate(for the particular durable) was used to determine how many years it would take a purchase of D_i to essentially 'disappear' from the stock (again assuming exponential decay). For instance, if δ_i = .8 then any investment in this good from one period past would contribute .2 (i.e., = $1-\delta_i$) of the original expenditure to the current stock; expenditure two periods ago would contribute $(1-\delta_i)^2$ or .04 of DE_{it-2};

---

[1]See J. Helliwell and Others, "Quarterly Estimates of Private Sector Wealth," Reserve Bank of Australia, August 1971, p. 21. The purpose of the Australian study (as in the RDX2 model) is to measure the replacement value of the net stock of assets.

and remaining from three periods back would be $(1-\delta_i)^3$ or .008 of that period's expenditure. Thus, an approximation to the 'initial' stock for any given starting point t, would be:

$$\sum_{t^*=1}^{3} \underset{\text{Expenditure}}{\overset{\text{Durable}}{DE_{i_{t-t^*}}}} (1-\delta_i)^{t^*}$$

which represents the sum of the undepreciated proportion of past investment expenditures. Note that the larger the δ_i we choose, the larger looms immediate past investment in forming our current stock.

3. The Durable Goods

a. Clothing

Although no reasonable second-hand market exists, we still treat clothing as a durable since our simple definition includes any good which is not completely consumed during the expenditure period (i.e. $0 \leq \delta_i < 1$). By this standard, an old pair of shoes still contribute some 'value' to the consumer's personal stock of clothing, whether he wears them or leaves them in his closet.

"The" depreciation rate for clothing is a difficult concept, since a man's brogues may be kept for six years whereas his wife may replace her entire wardrobe on the simple pretext that "fashions have changed" only seven months after the initial purchase. However, assuming *some* rationality, we have decided upon an annual rate of deterioration of .5 which implies a lifetime of about five to six years with a useful* lifetime of two to three years. This seems reasonable when we consider the wide variety of entries in the clothing category. After the real stock series was calculated (assuming $\delta_c = .5$),

---

*i.e., assuming there is (arbitrarily) at least 20-30% of the initial investment remaining.

different values for δ_c were tried, and the resultant 1969 stock values were too divergent from the assumed level; that is to say, the values exhibited either too rapid a deterioration (.6) or too much accumulation to seem correct (.3) and (.4) if we believe in $\delta_c = .5$; of course, this is all arbitrary.[2]

The expenditure series was extracted from Tables 53 and 54 of Canada's National Accounts (Personal Expenditure on Consumer Goods and Services in current and constant [1961] dollars respectively) and the implicit purchase price index was obtained by simple division of the current by the constant dollar values.

b. Motor Vehicles

Among consumer durables, motor vehicles command perhaps the most agreement with respect to depreciation rates. It is accepted by most authorities that an automobile depreciates anywhere from 25 to 30 per cent annually, and the "red book" of Australia and "blue book" of Canada (wholesale values of used cars) follow this guideline in valuating automobiles. While it is acknowledged that cars of different vintages (as would be expected) depreciate at different rates; the present paper assumes an "average" rate of 28 per cent over all vintages. To support this figure, we may look to several sources:

	SOURCE	ANNUAL δ RATE	
[22]	Reserve Bank of Australia (August 1971)	.22	+ one-half of quarterly rate upon new investment
[23]	Bank of Canada (Structure of RDX2)	.22	" "
[25]	Houthakker and Taylor	.25	
[21]	Hamburger	.28	

[2]We have not included the clothing stock series for alternative depreciation rates in the Appendix.

Hamburger (abcve) states that this rate "is within the range implied by the behaviour of prices in the used car market," and he suggests that Chow[3] and Friedman[4] provide verification. Another vote of support for this rate is the Canadian taxation system, under which a businessman may depreciate his car at 30% the first year and 25% the next two years.

The National Accounts entry includes motor vehicle parts as well as net (i.e., less trade-in value) used automobiles. Actual repair data are included in "automobile operating expenses and purchased transporation," but a search of the sources and notes revealed no way to decompose the latter into "repairs to automobiles," "operating expenses" (gasoline, oil, and possibly insurance), and "purchased transportation."

The historical National Accounts [13-502; pp. 157-58], discusses the procedure involved in the inclusion of only the personal use portion of car sales in the reported expenditure data.

The same source also mentions the coverage of "net" used automobiles:

> . . .the treatment of purchases of used motor vehicles by persons from *businesses* has been changed. The total amount paid by persons less the value of trade-ins is now included in personal expenditure on consumer goods and services.[5]

---

[3]G. Chow, "Statistical Demand Functions for Automobiles and Their Use for Forecasting," in A. Harberger (Ed.), *The Demand for Durable Goods*, Chicago, 1960 [21a].

[4]C. Friedman, "The Stock of Automobiles in the United States," *Survey of Current Business* (October, 1965), pp. 21-28.

[5]National Accounts (13-502), p. 103.

The base stock was estimated in the manner discussed in section A, using δ_{MV} = .28 and the constant dollar expenditure series from Table 54 of the National Accounts. It was necessary to accumulate depreciated investment from 11 years back (1935).

c. Other Durables

The concept of "other" durables arises as a residual category, namely, "durable goods" minus consumer expenditure on motor vehicles. This fact implies that the category encompasses a wide variety of goods. In fact, we see from the notes to Tables 53 and 54 (unpublished) that the category "durables" includes house trailers, [AUTOMOBILES], furniture, household appliances, radio and television sets, and sporting and wheeled goods. By definition, then, the residual includes all but automobiles. Such items as watches, clocks, jewellery, silverware, tools, garden equipment, and toys are *not* considered durables in the National Accounts.[6]

The most comprehensive discussion of "other durable goods" occurs in Helliwell *et al* [22], where the research went beyond regression equations and statistical tests, since an actual survey of households was conducted in order to obtain estimates for the average lifetimes of the goods in question. [Another reason for the difficulty in treating the category is the obscurity of the 'base stock' concept]. Notwithstanding the disclaimers, and recognizing the fact that the figures are still basically drawn from a hat," let us consider some results from the survey. The study

---

[6]These are, however, classified as "semi-durables."

considered such factors as the average age at scrapping and weights from a composite consumer price index in order to arrive at an "average" quarterly depreciation rate:

> (1) Electrical goods (washing machines, refrigerators, cookers, television sets, etc.) were found to be the highest depreciating goods in the category, with a quarterly rate of 6.29%;
>
> (2) Hardware and other such items were next with an estimated quarterly rate of 5.5%; and
>
> (3) Furniture and floor coverings (kitchen, lounge, recreation) were estimated to depreciate at a rate of 5.22% per quarter.

Applying the weights,[7] we arrive at an average quarterly rate of 5.75% for other durables. Accumulating for four quarters [i.e., $(1-\delta^{Q}_{OTH})^{4}$] yields an approximate annual δ_{OTH} of 21%.[8] This is the figure we use in this study.

In calculating a price index for this residual category we have used a weighted index under the assumption that

$$\underset{\text{DURABLES}}{\text{Price}^{*}_{\text{TOTAL}}} = W_{MV}P^{*}_{MV} + \underset{(1-W_{MV})}{\overset{}{\underset{\shortparallel}{W_{OTH}}}}P^{*}_{OTH}$$

where W_{MV} = a weight for motor vehicle expenditure; p^{*}_{MV} = motor vehicle purchase price; and p^{*}_{OTH} = other durables purchase price.

---

[7] Electrical (.43); Furniture (.32); and Hardware (.25).

[8] This seems reasonable relative to δ_{MV} of .28 since we would expect a chair in one's den to last somewhat longer than a motor vehicle on the freeway. Hamburger estimates a rate of .5 but (a) admits that it is probably too high and (b) reports that a likelihood ratio test yields a probability that the "true" rate on other durables is no greater than that on automobiles of greater than 30 per cent [21; p. 1137].

and solved for

$$P^*_{OTH} = \frac{P^*_{TOTAL} - W_{MV} P_{MV}}{1 - W_{MV}}$$

using as weights the expenditure share (in current dollars) $\frac{\text{\$ expenditure M.V.}}{\text{\$ expenditure durables}}$

The resultant price index for other durables appears at the end of this chapter. This is the P^*_{OTHDUR} used to calculate the rental price PDO. Professor J. Helliwell has pointed out that the national accounts expenditure series were not estimated consistently, since motor vehicle expenditure does not include sales tax while "other durable goods" does account for taxes. We have chosen not to attempt to correct for the tax, since one arbitrary tax rate (for example 5.5%) is inappropriate. In calculating, the base stock for Other Durables, we have accumulated investment expenditure (depreciated) for 16 years prior to 1946.

d. Housing

As mentioned above, the National Accounts treat consumer expenditure on housing as *business* capital formation, and the figures reported in the consumer sector are imputed rents, etc. from the stock of housing.[9]

Recently, the Housing and Building Permits Division of Statistics Canada made available to us some preliminary results on the housing stock, and these data will be incorporated into this study. However, it is valuable

---

[9] Consumer expenditures include actual rents paid and rents imputed to various non-reported sources.

to first discuss other various sources of data for this very important sector.

One of the features of this study is the inclusion of land as a consumer durable. Since land and structure are complementary, it is surprising that no other studies have made an attempt (albeit crude) to treat the demand for residential land.

The Central Mortage and Housing Corporation (CMHC) publishes an annual entitled *Canadian Housing Statistics*, which presents comprehensive information on new starts, completions, method of financing and related data for housing. The main body of data on costs (land, construction, mortgage, etc.) focuses on new single-detached dwellings financed under the National Housing Act (NHA). (Since the NHA finances about 60 per cent of all starts in Canada, these data are reasonably representative). Less frequent mention is made of duplexes and multiple dwellings since the main concern of this publication is single-detached dwellings. These other categories are, however, treated.

(i) <u>Residential Land</u>. While an index of land cost and indexes of house size are available from the CMHC, no justification exists for relating the *cost* of land to a particular *size* of lot. Municipal Real Estate Boards,[10] or at least the Vancouver version, estimate data on site values for apartments, industrial land values, and commercial land values[11]

---

[10] See, for example, *Real Estate Trends in Metropolitan Vancouver*, 1971, Statistical and Survey Committee, Real Estate Board of Greater Vancouver.

[11] *Ibid.*, p. B5, C7, and C14 respectively.

per square foot. These data, however, are not useful unless we have reliable estimates on the *quantity* of square feet. Municipal Statistics (published by most provincial statistical bureaus) give sparse data on total acreage and assessed value of land sites for taxation purposes; but no breakdown is given for assessments for business and private purposes. Thus, while we can calculate that an "acre" of land in British Columbia was assessed *on the average* at $2184 in 1970, and $39,608 in Greater Vancouver, we can go no further.

The only comprehensive treatment of land in its different uses is in Goldsmith [19]; but his work is limited to wealth estimates for the U.S.A.

Our early estimates for the stock of residential land come from Firestone [14]. The Firestone series (in 1939 dollars) is available to 1953, and thus it was necessary to derive some estimates for later years. Firestone's series is based on an interpolation of the ratio of land acquisition cost (in 1944) to total cost of the dwelling (including land).* However, since no other attempts have been made, we have chosen to use this series as a starting point. To extend the residential land series, we have assumed a simple relationship between different types of dwellings and the size of lot each requires. Thus, for example, if an apartment block contains 20 dwelling units and covers a lot of 10,000 square feet, then each suite "requires" 500 square feet of land. Similarly, if a single-detached dwelling is erected on a 3,000 square foot lot, then we would say that (if we let the 500 square feet represent one unit of land stock) the house uses six land units.

---

*Since the 1944 ratio is *not* representative of all time periods, this is clearly *not* a satisfactory estimate.

Using these "land units," we may approximate a series of *additions* to the land stock (which links to Firestone's) in the following manner:

$$LAND_t - LAND_{t-1} = k\{L_1 \cdot (N_1 - D) + L_2 \cdot (N_2) + L_3 \cdot (N_3)\}$$

where

D = number of demolitions in year t;

L_i = number of "land units" required by dwellings of type i;

N_1 = number of new starts of single detached dwellings in year t;

N_2 = number of new row and duplex starts;

N_3 = number of new apartments starts.

In order to solve for k, we must first estimate L_1, L_2, L_3 using Firestone's 1952 and 1953 data, and then we may build up a series of additions (in 1939 dollars) to his land stock.

The *minimum* legal lot size requirements (to obtain a City of Vancouver building permit) are:

APARTMENTS (Prior to Sept. 1, 1965):	5,400 sq. ft.
(Since Sept. 1, 1965):	6,000 sq. ft.
LOW-DENSITY MULTIPLE HOUSING:	10,000 sq. ft.
DUPLEX:	3,800 (- 4,800) sq. ft.
SINGLE-DETACHED:	3,300 (- 4,800) sq. ft.

(These, of course, are only broad approximations, and the estimates to be derived should be considered the same). Using Vancouver land requirements and assuming the "typical" apartment block prior to the 1960's (in fact, we arbitrarily set 1962 to be "the" year the average apartment block grew) contained 24 dwellings and the post-1962 apartment contained 48 dwellings,

we make use of the following information:

DWELLING TYPE	AVERAGE LOT (sq. ft.)	SQUARE FEET OF LAND PER DWELLING
SINGLE-DETACHED	4,300	4,300 = 34.4 units
DUPLEX OR ROW	4,300	2,150 = 17.2 units
APARTMENT BLOCK: (24 dwellings) 1948-1962	5,400 sq. ft.	225 = 1.8 units
(48 dwellings) 1963-1969	6,000 sq. ft.	125 = 1 unit

Dwelling starts of different types come from the Central Mortgage and Housing Corporation and estimates of demolitions are found in Urquhart and Buckley [DATA (22)]. It was necessary to assume an average of 9000 demolitions from 1961 to 1969.

Thus, our "land increments series" is built up by solving

$$\underset{1952 \text{ to } 1953}{\Delta \text{LAND}} = k[34.4(N1-D) + 17.2(N2) + 1.8(N3)]$$

for k and then applying the value obtained to the dwelling starts series (in 1963 the 1.8 is replaced by 1.0). We adjust single-detached only (for disappearances) since this is the most likely to be demolished.

Once this series (ΔLAND) was constructed, we linked Firestone's (implicit) price index with a CMHC land price index and thus converted the 1939 dollar land stock to 1961 dollars. The resultant series appears at the end of this chapter.

(ii) Dwellings. In tabulating "the" stock, we must recognize that there are many methods of taking account, and our problem is that of reconciling these disparate measures into a common yardstick. The main bases (once we have sorted out single-detached, row, duplex, and apartment units) are square feet and rooms. Insofar as rented quarters are concerned, the last all-inclusive study in Canada was published in 1959[12] and stated this about a standard of measurement:

> Size can be defined in such terms as number of square feet of living space, number of rooms, or number of bedrooms. The latter is now widely accepted as a meaningful description of size and has been adapted in these rent tabulations. Most dwelling units offered for rent are described in terms of bachelor apartments or 1,2, 3, or more bedroom apartments or houses. Tabulation by size of dwelling unit is on this basis.[13]

Since 1953, there has been an annual publication[14] which estimates the total number of rooms (and dwelling units) in the Canadian housing sector. These data are tabulated in Usher [36] where the total annual stock of rooms of dwelling space is reported for units from 1 to 9+ rooms. Some concepts from the original source:

> Dwelling -- a dwelling is defined as a structurally separate set of living premises with private entrance from outside the building, or from a common hallway or stairway inside.

---

[12] Canada, Statistics Canada (formerly the Dominion Bureau of Statistics, and henceforth referred to as DBS), Catalogue No. (62-519).

[13] *Ibid.*, p. 10.

[14] DBS (64-202).

Household -- a person or a group of persons occupying one dwelling unit is defined as a "household." The number of households will therefore be equal to the number of occupied dwellings.

Apartment or Flat - includes:

(a) dwelling units in apartment blocks or apartment hotels;
(b) suites in duplexes or triplexes (i.e., where the division between dwelling units is horizontal);
(c) suites in structurally converted houses;
(d) living quarters located above, or in the rear of stores, restaurants, garages, or other business premises;
(e) Janitors' quarters in schools, churches, warehouses, etc.; and
(f) private quarters for employees in hospitals or other types of institutions.[15]

Pickett [29] calculated early estimates for the housing stock (1871-1921) using decennial census information on occupied, vacant, and incomplete dwellings. He began on the premise that "it is possible to derive estimates of decade changes in housing stock and, by the application of related economic indicators, to distribute these changes on an annual basis."[16] Among his assumptions were the following:

(a) before 1911, apartments were of negligible importance;

(b) in the early years, since Canada was an expanding frontier society, temporary dwellings were important;

(c) the 'economic indicator' which he uses to distribute decennial changes to annual changes is a series on the consumption of window glass:

---

[15] DBS (64-202), 1971, Pp. 6-7.

[16] Pickett, "Residential Capital Formation in Canada, 1871-1921," *Canadian Journal of Economics and Political Science*, Vol. 29-1 (February, 1963), p. 41.

> Thus, on a thought no more profound than that houses have windows, it was decided to secure information on window glass imports and consumption; make this information consistent with certain reasonable assumptions about building activity, and to allocate the total number of permanent houses completed in the successive decades to an annual basis by means of [this] series. . .[17]

Pickett's value estimates are based on 1921 home values in Firestone (*Residential Real Estate in Canada*). Interestingly, Pickett's early estimates are to a surprising extent statistically consistent with official estimates for later years.

More recently, Marion Steele [34],[18] has treated housing starts as a consumer decision, and the stock estimates she derives are consistent with recent work being done at Statistics Canada. In fact, Steele is involved in the government's housing stock project, so we may expect similarities in the results.

The recent estimates by Statistics Canada of housing capital stock are "estimates of fixed reproducible residential stock at market value."[19] This stock includes all non-collective dwellings intended for year-round occupancy, and not owned by the government. Included are such related items as permanently-fixed equipment and garages. More important, "capital formation

---

[17] Pickett, *op. cit.*, p. 45.

[18] "Determinants of Owner-Occupied Dwelling Starts, Canada, 1921-1940," University of Guelph, March, 1972.

[19] "Estimates of Residential Capital Stock and Flows, Methodology," Housing and Building Permits Section, Statistics Canada, 1972, p. 1.

contributing to the stock may take the form of major alterations or improvements to existing dwellings."[20]

Market values from the 1941 Census are used as a benchmark in the stock estimates, and depreciation on the stock was taken at two per cent annually in a perpetual inventory framework, but again gross capital formation is depreciated at one-half the annual rate in the current year. Thus, it was necessary to recalculate the (real) stock from the beginning of the series (1921) using our geometric depreciation with *no* current period depreciation on new capital. Another minor change in this study is the use of $\delta_{HOUS} = .03$, since this will mean that 80 per cent of the initial value is depreciated in 50 years as opposed to 80 per cent in 75 years at an annual rate of .02.[21] After half a century, there is great probability that a house will end in the demolished group.

The Statistics Canada data series excludes real estate commission (although the National Accounts include it). It is clear that such a component *could* be considered part of capital formation, since commission is paid to facilitate the transfer of stock. The figures are available in current and constant dollars, and thus the resultant implicit stock price index is consistent with the other goods in our model.

---

[20]*Loc. cit.*

[21]Goldsmith [19] suggests that 50 years is a good approximation to "the" lifetime of a house.

(iii) Property Taxes. Since we will be calculating a user cost for housing and land, we must account for property taxes, which are a real cost to the consumer. We will assume the same property tax rate on residential land and structures.

In fact, what we basically do is (i) to estimate a stock of business land series (LB) in current dollars to go with our residential land (LR), residential structures (SR), and a business structures series (SB) estimated by Professor A. D. Woodland;* (ii) take the value of property taxes from the National Accounts as a ratio (annually) to the total current dollar value of LR + SR + γ(LB + SB) where γ is a revaluation factor of business property for taxation purposes. A detailed description appears in Table IV-4 in the Appendix.

B. THE RATE OF INTEREST RELEVANT TO CONSUMER EXPENDITURE DECISIONS

In choosing a discount rate for purchases of consumer durables, we must, of course, confront the ever-present problem of the discrepancy between borrowing rates and lending rates; a problem which often confuses empirical analyses. As a rough approximation, the 1971 Canadian consumer would have to pay anything from 7¼% (fully-secured bank loan) to 12% (bank loan for consumer expenditure secured by the asset itself) to 18% (the maximum allowable under most provincial Small Loans Acts, and the annual rate charged by all department stores and credit card organizations on outstanding balances)

---

*This is part of Professor Woodland's current research project on production functions for Canadian industries.

in order to "borrow" from his future labour income to finance current expenditure.

The Small Loans Act rate is non market-determined; rather, it is a legislated "ceiling" on consumer credit charges. Clearly, most organizations adhere very closely to the "ceiling" and rarely will an interest charge of *less than* 16% be in effect under such an arrangement.

Diewert [(10, p. 26)] calculates a rate of return after taxes on corporate profits and after income taxes on non-labour income, i.e.,

$$R = r - (T_C + T_K) / P_K \cdot K$$

r	T_C	T_K	P_K	K
Before Taxes Gross Private Rate of Return	*Corporate Taxes*	*Personal Taxes Attributed To Capital Income*	*Price Index for Capital Stock*	*Beginning of Year Real Capital Stock*

If we assume that R is the net rate of return the consumer *could have earned* (i.e., the opportunity cost) had he not purchased the durable, we also assume that the funds were available for either use. In such a case, a bond yield series (short-to-medium term to maturity) would serve as a reasonable proxy for the rate of return on alternative investments; for example, the yield on Government of Canada bonds with one to three years left to maturity may well be the appropriate rate facing the 'typical' consumer with a shorter planning horizon.

The dichotomy is well-illustrated in two works by the same author;[22] however, the probable reason for this is that the latter work is more theoretical.

---

[22] W. E. Diewert [10, p. 8]. "One of the main simplifying assumptions in our model is that there exists only a single financial asset, denominated

Hamburger [21] argues that nominal bond and savings account interest rates are very relevant in explaining expenditures on consumer durables, although his results indicate that bond yields have their maximum effect on consumer expenditures after a four-to-six quarter lag. In fact, Hamburger suggests that the (relative) length of these lags suggests that (in his case, Moody's Aaa rate on long-term corporate bonds) "is acting primarily as a proxy for the rates charged on consumer credit, rather than as a measure of the yields available on marketable financial assets."[23] He concludes:

> (a) marketable financial assets are relatively poor substitutes for consumer durable goods; and
>
> (b) the estimated lags may be more appropriately viewed as estimates of the reaction times of the suppliers of consumer credit than those of consumers. That is, they serve as indicators of the speed with which the rates charged on consumer credit adjust to changes in open-market rates.[24]

Another writer (W. Weber) in intertemporal consumer theory propose that r be an average effective interest rate or an *index* of interest *rates* which the consumer faces.[25]

What the above discussion suggests is (i) that there is no single unambiguous "rate" which the consumer faces; and that (ii) under different

---

in units of the aggregate capital stock, which is expected to yield an own rate of return equal to R_t during period t. In [11; p. 63] r is defined as the one-period interest rate at which the consumer can borrow.

[23] Hamburger, *op. cit.*, p. 1139.

[24] *Loc. cit.*

[25] "The Effect of Interest Rates on Aggregate Consumption," *American Economic Review* (September, 1970), p. 593.

assumptions, the opportunity cost of purchasing a consumer durable may well be either a borrowing rate (how much he must pay for the funds), a lending rate (how much he could conceivably earn from a different allocation of the funds), or some rate which lies between the two.

One of the existing rate of return series is available as 'RHO' from the Bank of Canada RDX2 model. The series represents (an estimate for) the nominal supply price of capital, and is available from 1952 quarterly. This series, however, would be difficult to extend back to 1946.

The interest rates actually used were different* for each of the durables. For clothing, we used an average yield on Government of Canada securities with five years to maturity; for motor vehicles and other durables, the respective rates of interest charged to finance their purchases, and for housing and land the maximum (annual average) rate on NHA-insured mortgages. The sources of these series appear in Table IV-4 in the Appendix.

C. RENTAL PRICES

The 'rental' price is equivalent to the user cost for one unit of a durable good. (If the consumer purchases a unit of good k, call p_k the price he charges himself for the *flow* of services he has used).[26]

---

[26] Diewert, "Intertemporal Consumer Theory," *op. cit.*, p. 9: "To form the rental price (or user cost) for the services of one unit for the nth good during period t, we imagine that the consumer purchases the good during period t and then sells it during the following period (possibly to himself). Then the discounted expected rental price for the nth consumer good during period t is given by the discounted cost of the purchase of the nth good during period t minus the discounted resale value of the depreciated good during period t+1. . .

*Since it is unlikely one would borrow to finance the purchase of clothing the chosen interest rate is a lending rate. For the other durables, where borrowing is more likely, we have chosen the cost of credit (borrowing rates).

We will make the simplifying assumption of static expectations so that our consumer expects current prices and interest rates to prevail. In this framework, then

$$(p^*_{kt} - p_{kt}) \cdot (1 + R^*_{kt}) = (1 - \delta_k)p^*_{kt+1}$$

where

p^* is a stock (purchase) price; and

p the rental price.

Since we have assumed static expectations, p^*_{kt+1} is expected to be equal to p^*_{kt}; thus:

$$(p^*_{kt} - p_{kt}) \cdot (1 + R^*_{kt}) = (1 - \delta_k)p^*_{kt}$$

This reduces to a formula for p_{kt}:

$$p^*_{kt}(1 + R^*_{kt} - 1 + \delta_k) = p_{kt}(1 + R^*_{kt})$$

i.e.,

$$p_{kt} = \frac{p^*_{kt}(\delta_k + R^*_{kt})}{(1 + R^*_{kt})}$$

The rental prices for clothing, automobiles, and other durables are calculated in this manner. Those for housing and land, however, are slightly modified, since it is important to account for the effects of a property tax. Thus, if we let t^* represent the rate of property tax, then the rental price for housing becomes:

$$p_{Ht} = p^*_{Ht}(1+t^*) - \frac{(1-\delta_H)p^*_{Ht}}{1+R_{Ht}}$$

and that of land (where there is no depreciation):

$$p_{LT} = p^{*}_{LT}(1+t^{*}) - \frac{p^{*}_{Lt}}{1+R_{Ht}}$$

The Role of Expectations

Clearly, the consumer has expectations for future prices, wage rates, and interest rates. Since the rental price is formed from a stock price expected to prevail, and since it is unlikely that our consumer has perfect foresight, we have assumed a model with static expectations. Theoretically,"we could make future expected prices a distributed lag of past prices and then use the derived demand equations to simultaneously estimate the parameters of preferences as well as the distributed lag."[27] This would lead to very complicated estimation procedures.

D. NONDURABLES

The series for food, alcoholic beverages, and tobacco products are directly from the National Accounts (Tables 53 and 54, items 2, 3, and 4). The implicit price indexes were calculated by dividing the current dollar expenditure by constant (1961) dollar expenditure.

E. SERVICES

We have broken the National Accounts "services" into (a) medical care and health services; and (b) other services. The 'other' category

---

[27]Diewert, "Intertemporal Consumer Theory," *op. cit.*, p. 12.

includes recreation, telephone, cultural services, financial, legal, and several others. Again, since 'other' is a residual category, it was necessary to calculate a 'weighted' price index for 'other.' The procedure was identical to that used in calculating a purchase price index for 'other durables.' The price and quantity series are tabulated at the end of the chapter.

1. Medical Care and Health Services

Beginning in 1961, the major proportion of expenditures on hospitals appears in the government sector. To correct for this in consumer expenditures, we have added in the government's expenditure on hospitals to consumer hospital expenditure for the years 1961 to 1969. The constant dollar series was obtained by deflating current dollar value by a physician services price index.[28] The data were taken from *The National Finances 1970-71* [DATA (21); p. 10] and Table 52 of the National Accounts (government transfers). The latter came in for 1968 and 1969, where it was necessary to include federal [transfer] expenditure to the provinces for medicare in "physician services,"[29] which is one of the components of "medical care and health services." Thus, the medical series (and consequently the other services series) are *not* the same as those appearing in the National Accounts.

---

[28] I am indebted to Professor T. J. Wales for pointing out this problem.

[29] Professor D. Usher made this information available.

F. THE SUPPLY OF LABOUR

We take the Diewert-Woodland total wage bill (over all industries) in current dollars (call it W) and calculate an "average" hourly wage rate thus:

1. Statistics Canada -- unpublished data on average weekly hours worked (cf. Table IV-9) times 50 weeks = average annual hours worked per person.

2. To obtain an annual hours bill, we multiply (1) by employment (tabulated in [20]);

3. Finally, we divide W by (2) to obtain an hourly wage rate (cf. Table IV-10 for this and all subsequent labour data).

If we divide the total hours bill by our population series we arrive at per capita hours worked (L). In this manner, we have a consumer theory approach to labour force participation rates. Assuming that the *average* worker would probably not work more than 60 hours per week for 52 weeks we may define leisure hours as $\ell \equiv 3120 - L$. We have assumed that physiologically required time (i.e., sleeping) is *not* leisure.

Discussion. Again, with the supply of labour, we face several problems. There is, of course, the aggregation problem. More seriously, there is the institutional constraint that the consumer-worker really does not have a great deal of choice insofar as *choosing* how many hours he will work. This is not, in the real sense, a decision variable. Unfortunately, there is no way to circumvent this difficulty. One further complication arises in the tax rate (see below), since inherent in the progressive taxation system is a higher tax bracket for more hours worked (i.e., higher income). Again, we cannot capture the progressivity of the tax structure in our model.

1. Tax on Labour Earnings

We make only a crude adjustment for income and other taxes on labour earnings. Our tax rate (τ) is simply the ratio of (total direct taxes *less* succession duties and estate taxes) = TD to Personal Income (PI). The taxes are found in Table 44 of the National Accounts and Personal Income is from Table 4 of the same source. Thus τ=TD/PI and $w(1-\tau)$ is the price of time. The series TD and PI appear in Table IV-11 of the Appendix.

Thus, we have developed a reasonably broad data base for consumption expenditure in post-war Canada. In Chapters V and VI we will examine the results of our analysis carried out on these data.

A P P E N D I X

TO

CHAPTER IV

TABLE IV-1

AGGREGATE STOCK SERIES

CONSTANT (1961) $MILLIONS
BASE YEAR = 1946

YEAR	CLOTHING $\delta = .50$ INITIAL STOCK=1423.92 PURCHASES	X_i = STOCK	MOTOR VEHICLES $\delta = .28$ INITIAL STOCK=237.92 PURCHASES	X_i = STOCK	OTHER DURABLES $\delta = .21$ INITIAL STOCK=1288.17 PURCHASES	X_i = STOCK	HOUSING $\delta = .03$ INITIAL STOCK=11411.59 PURCHASES	X_i = STOCK	RESIDENTIAL LAND $\delta = 0.0$ INITIAL STOCK=3921.10 PURCHASES	X_i = STOCK
1946	1676.00	2387.96	145.00	316.30	631.00	1648.65	678.39	11747.63	109.31	4030.41
1947	1653.00	2846.98	312.00	539.74	761.00	2063.43	668.89	12064.10	129.79	4160.21
1948	1511.00	2934.49	267.00	655.61	747.00	2377.11	850.39	12552.57	129.91	4290.12
1949	1498.00	2965.24	461.00	933.04	831.00	2708.92	990.59	13166.59	148.92	4439.04
1950	1510.00	2992.62	753.00	1424.79	863.00	3003.04	1125.29	13896.89	272.00	4711.04
1951	1473.00	2969.31	663.00	1688.84	786.00	3158.40	839.20	14319.18	149.51	4860.55
1952	1606.00	3090.65	825.00	2040.97	904.00	3399.14	943.39	14833.00	155.60	5016.16
1953	1667.00	3212.32	939.00	2408.49	1115.00	3800.32	1250.50	15638.51	97.23	5113.39
1954	1682.00	3288.16	861.00	2595.11	1187.00	4189.25	1424.79	16594.15	107.58	5220.98
1955	1784.00	3428.08	1125.00	2993.48	1450.00	4759.50	1778.79	17875.12	138.91	5359.89
1956	1914.00	3628.04	1202.00	3357.30	1562.00	5322.00	1776.79	19115.66	125.29	5485.18
1957	1986.00	3800.02	1101.00	3518.26	1553.00	5757.38	1540.39	20082.59	114.49	5599.67
1958	2062.00	3962.01	1137.00	3670.14	1619.00	6167.33	1937.39	21417.51	147.31	5746.98
1959	2155.00	4136.00	1192.00	3834.50	1736.00	6608.19	1867.39	22642.38	129.05	5876.04
1960	2242.00	4310.00	1184.00	3944.84	1723.00	6943.46	1521.20	23484.30	94.09	5970.13
1961	2276.00	4431.00	1227.00	4067.28	1856.00	7341.33	1488.00	24267.77	108.44	6078.57
1962	2352.00	4567.50	1416.00	4344.44	2063.00	7862.65	1549.59	25089.33	107.00	6185.58
1963	2393.00	4676.75	1621.00	4749.00	2123.00	8334.49	1613.70	25950.35	108.16	6293.74
1964	2477.00	4815.37	1871.00	5290.27	2301.00	8885.25	1870.79	27042.64	109.81	6403.55
1965	2575.00	4982.68	2162.00	5970.99	2464.00	9483.34	1970.59	28201.96	107.44	6510.99
1966	2625.00	5116.34	2246.00	6545.11	2617.00	10108.84	1813.50	29169.40	98.86	6609.85
1967	2690.00	5248.17	2279.00	6991.48	2644.00	10629.98	1873.00	30167.32	106.04	6715.89
1968	2772.00	5396.08	2469.00	7502.86	2768.00	11165.68	2183.70	31446.00	111.94	6827.83
1969	2890.00	5588.04	2525.00	7927.06	3039.00	11859.89	2522.70	33025.31	118.66	6946.49

Note: In [40; p. 230], reference is made to a Department of Agriculture memo estimating the total area of residential land in Canada to be 2,160,000 acres in 1963. This (using our real stock in 1961 dollars) works out to be $2,913.77/acre in constant dollars and $3,328.98/acre in current dollars.

TABLE IV-2

CANADA: POPULATION AND EMPLOYMENT
(Thousands)

YEAR	POPULATION 15+*	EMPLOYMENT
1946	8,871.6	4,666
1947	9,012.6	4,832
1948	9,144.7	4,875
1949	9,508.9	4,916
1950	9,641.5	4,978
1951	9,758.7	5,098
1952	10,006.3	5,161
1953	10,216.9	5,237
1954	10,452.3	5,244
1955	10,659.1	5,364
1956	10,855.5	5,586
1957	11,153.4	5,726
1958	11,394.8	5,694
1959	11,625.3	5,856
1960	11,840.0	5,956
1961	12,046.4	6,050
1962	12,273.3	6,220
1963	12,513.1	6,366
1964	12,791.7	6,599
1965	13,087.7	6,848
1966	13,423.2	7,143
1967	13,811.5	7,364
1968	14,178.5	7,522
1969	14,542.4	7,764

*15 years of age and older.

Sources:

1. Population -- DBS (91-201); (91-202); (91-511).

2. Employment -- Labour Force Survey, Special Table 3c; tabulated in [20].

TABLE IV-3

DURABLES: PURCHASE PRICE INDEXES

YEAR	CLOTHING	MOTOR VEHICLES	OTHER DURABLES	HOUSING	LAND
1946	0.6122	0.5586	0.6534	0.5410	0.2497
1947	0.6939	0.6474	0.8428	0.6100	0.2787
1948	0.8432	0.7715	0.8263	0.7150	0.3188
1949	0.8785	0.8113	0.8489	0.7450	0.3310
1950	0.8821	0.8207	0.8852	0.7820	0.3699
1951	0.9830	0.9366	1.0018	0.9020	0.4027
1952	0.9875	0.9527	1.0177	0.9179	0.4542
1953	0.9784	0.9361	1.0032	0.9260	0.4600
1954	0.9721	0.9408	0.9865	0.9155	0.6483
1955	0.9608	0.8684	0.9632	0.9360	0.6990
1956	0.9666	0.8827	0.9670	0.9500	0.7782
1957	0.9668	0.9655	0.9825	0.9770	0.8685
1958	0.9763	0.9815	1.0036	0.9740	0.9496
1959	0.9759	1.0252	1.0062	0.9730	0.9734
1960	0.9848	1.0287	1.0050	0.9919	0.9504
1961	1.0000	1.0000	1.0000	1.0000	1.0000
1962	1.0098	0.9859	0.9990	0.9970	1.0695
1963	1.0372	0.9858	1.0073	1.0200	1.1425
1964	1.0622	0.9578	1.0130	1.0599	1.1844
1965	1.0796	0.9500	1.0192	1.1209	1.1894
1966	1.1188	0.9328	1.0401	1.1960	1.3374
1967	1.1729	0.9491	1.0859	1.2630	1.3758
1968	1.2042	0.9660	1.1175	1.2770	1.4396
1969	1.2384	0.9711	1.1297	1.3360	1.6145

Source: National Accounts, Income and Expenditure, Tables 53 and 54; Land -- CMHC Index of Residential Land Costs.

TABLE IV-4

DURABLES: DISCOUNT RATES AND AVERAGE PROPERTY TAX RATE

YEAR	RSBY	RMV	RCFP	RNHA	PTAX
1946	.0169	.1300	.1560	.0450	.0638
1947	.0175	.1300	.1560	.0450	.0599
1948	.0227	.1300	.1560	.0450	.0564
1949	.0224	.1300	.1560	.0450	.0566
1950	.0222	.1300	.1560	.0450	.0573
1951	.0258	.1300	.1560	.0500	.0618
1952	.0323	.1300	.1560	.0525	.0643
1953	.0344	.1500	.1690	.0525	.0653
1954	.0271	.1485	.1690	.0550	.0674
1955	.0275	.1530	.1860	.0525	.0678
1956	.0372	.1580	.2010	.0550	.0714
1957	.0460	.1580	.2010	.0600	.0746
1958	.0347	.1580	.1860	.0600	.0752
1959	.0490	.1585	.1860	.0606	.0818
1960	.0455	.1585	.1860	.0675	.0848
1961	.0438	.1585	.1965	.0673	.0874
1962	.0456	.1585	.1985	.0650	.0898
1963	.0448	.1585	.1985	.0635	.0874
1964	.0471	.1490	.1985	.0625	.0871
1965	.0488	.1490	.1955	.0625	.0848
1966	.0555	.1395	.1995	.0692	.0822
1967	.0561	.1395	.2020	.0744	.0842
1968	.0668	.1390	.2010	.0884	.0894
1969	.0762	.1565	.2095	.0938	.0871

Sources:

CLOTHING: RSBY --yield on Government of Canada bonds with five years to maturity.

1946-1950: Urquhart and Buckley (Data [22]); linked to 1951-1969: RDX2 [23 -- DATATAPE] SERIES RMS.

Sources (continued)

MOTOR VEHICLES: RMV -- 1972 Canadian Consumer Credit Factbook (DATA[ ])

1949-1952: Low from 1953
1953-1969: Mean of high and low rates

Taken from Federated Council of Sales Finance Companies Survey of Ten Largest Companies.

OTHER DURABLES: RCFP -- as above for Motor Vehicles.

HOUSING AND LAND: (a) RNHA -- ceiling on NHA mortgages (average of 12 months); Central Mortgage and Housing Corporation (DATA [19] and [20]).

(b) PTAX -- see Notes (i)

Notes to Table IV-4

(i) Property Tax Rate (PTAX)

The first step was to estimate a business land series (SB) in the following manner:

1. From [40; p. 230] we know that in 1963 there were approximately 2,160,000 acres of residential land and that commercial and industrial land is in a 1:6 ratio to residential land. This yields an approximate 360,000 acres of business land in 1963.

2. *Assume* business land = α real stock of business structures and estimate α for 1963.

3. We thus may generate a business land series (in acres) throughout our sample period; we value this series at (the 1961 dollar value of land) x (land price index) to arrive at a current dollar value of business land.

4. We then revalue business land and structures in the following manner: Steele [34] reports that the 1940 tax rate on LR + SR was 3.09%. This is calculated from the 1941 Census by taking the ratio of property taxes to the value of property assessed for taxation.

5. We may then calculate the residential property tax bill in 1940 as .0309 (SR + LR) = t_1.

6. Assuming land and structures pay the same rate in the business sector, we define a valuation factor γ in this way:

$$\text{TOTAL PROPERTY TAXES} - T_1 \equiv .0309\gamma\ (SB + LB)$$

7. Then the annual tax rate on LR and SR, i.e.,

$$t^* = \frac{\text{TOTAL PROPERTY TAXES (Table 46 - NATIONAL ACCOUNTS)}}{\{LR + SR + \gamma(LB + SB)\}}$$

(ii) Estimated Business Land Series = α SB (acres)

1940	142,287	1957	254,509
1946	144,988	1958	272,724
1947	149,799	1959	287,840
1948	156,138	1960	304,907
1949	163,733	1961	322,297
1950	171,646	1962	338,963
1951	180,190	1963	360,000
1952	190,204	1964	380,719
1953	199,909	1965	402,328
1954	212,084	1966	429,060
1955	224,251	1967	461,651
1956	237,554	1968	490,170
		1969	515,479

TABLE IV-5

DURABLES: RENTAL PRICES

YEAR	PCL (1)	PMV (2)	PDO (3)	PHOUS (4)	PL (5)
1946	0.3112	0.2027	0.2069	0.0733	0.0267
1947	0.3529	0.2349	0.2668	0.0803	0.0287
1948	0.4310	0.2799	0.2616	0.0916	0.0317
1949	0.4489	0.2944	0.2688	0.0956	0.0330
1950	0.4506	0.2978	0.2803	0.1009	0.0371
1951	0.5039	0.3398	0.3172	0.1245	0.0441
1952	0.5092	0.3457	0.3222	0.1310	0.0519
1953	0.5055	0.3500	0.3252	0.1331	0.0530
1954	0.4989	0.3510	0.3198	0.1355	0.0775
1955	0.4933	0.3261	0.3216	0.1368	0.0823
1956	0.5006	0.3339	0.3309	0.1444	0.0961
1957	0.5047	0.3652	0.3362	0.1558	0.1139
1958	0.5045	0.3712	0.3351	0.1559	0.1252
1959	0.5107	0.3880	0.3360	0.1627	0.1352
1960	0.5138	0.3894	0.3356	0.1747	0.1407
1961	0.5210	0.3785	0.3397	0.1786	0.1505
1962	0.5269	0.3732	0.3405	0.1785	0.1613
1963	0.5408	0.3731	0.3433	0.1788	0.1681
1964	0.5550	0.3576	0.3453	0.1846	0.1728
1965	0.5649	0.3547	0.3457	0.1926	0.1708
1966	0.5888	0.3434	0.3551	0.2093	0.1965
1967	0.6176	0.3494	0.3722	0.2291	0.2111
1968	0.6398	0.3554	0.3824	0.2531	0.2456
1969	0.6630	0.3665	0.3918	0.2676	0.2791

(1) Clothing
(2) Motor Vehicles
(3) Other Durables
(4) Housing
(5) Land

TABLE IV-6

NONDURABLES: PURCHASE PRICE INDEXES

YEAR	PFOOD (1)	PALC (2)	PTOB (3)	PMED (4)	POSERV (5)
1946	0.5757	0.8034	0.8108	0.5073	0.5607
1947	0.6517	0.8079	0.8484	0.5296	0.5961
1948	0.7986	0.8282	0.8915	0.5946	0.6387
1949	0.8132	0.8265	0.9074	0.6226	0.6699
1950	0.8296	0.8447	0.9295	0.6319	0.7120
1951	0.9534	0.9169	1.0217	0.6911	0.7607
1952	0.9463	0.9165	1.0508	0.7495	0.7934
1953	0.9132	0.9178	0.9264	0.7744	0.8214
1954	0.9094	0.9198	0.9147	0.8107	0.8492
1955	0.9094	0.9212	0.9160	0.8335	0.8697
1956	0.9201	0.9236	0.9165	0.8633	0.8930
1957	0.9571	0.9501	0.9164	0.9115	0.9198
1958	0.9954	0.9638	0.9167	0.9463	0.9493
1959	0.9874	0.9819	0.9785	0.9746	0.9675
1960	0.9903	0.9923	0.9957	0.9926	0.9842
1961	1.0000	1.0000	1.0000	1.0000	1.0000
1962	1.0207	1.0205	1.0038	1.0237	1.0221
1963	1.0521	1.0226	1.0038	1.0456	1.0389
1964	1.0613	1.0508	1.0124	1.0673	1.0629
1965	1.0949	1.0602	1.0463	1.0965	1.0964
1966	1.1601	1.0741	1.0897	1.1279	1.1382
1967	1.1635	1.0924	1.1333	1.1990	1.1868
1968	1.2051	1.1718	1.2703	1.2354	1.2460
1969	1.2376	1.2121	1.3342	1.2872	1.3331

Price index series for nondurables and services:

(1) food
(2) alcoholic beverages
(3) tobacco products
(4) medical care and health services
(5) other services

TABLE IV-7

PER CAPITA QUANTITY SERIES

YEAR	FOOD	ALCOHOL	TOBACCO	HEALTH	OTHER SERVICES	CLOTHING	MOTOR VEHICLES	OTHER DURABLES	HOUSING	LAND
1946	373.773	53.315	45.877	77.102	453.531	269.169	35.654	185.835	1324.184	454.306
1947	374.052	58.918	45.382	82.337	454.359	315.889	59.887	228.950	1338.582	461.599
1948	349.584	59.813	46.366	80.920	450.971	320.895	71.693	259.944	1372.661	469.137
1949	334.944	61.203	46.590	81.922	470.328	311.839	98.123	284.883	1384.659	466.830
1950	352.937	62.130	47.089	86.500	482.902	310.390	147.777	311.471	1441.362	488.621
1951	351.571	61.691	42.425	88.223	507.042	304.273	173.061	323.650	1467.324	498.074
1952	355.583	68.260	43.273	90.937	542.638	308.871	203.969	339.700	1482.366	501.300
1953	364.092	69.105	47.861	94.161	563.025	314.413	235.737	371.964	1530.650	500.484
1954	368.741	68.026	48.218	98.068	572.775	314.588	248.282	400.796	1587.607	499.505
1955	374.998	71.391	49.161	101.977	594.053	321.611	280.838	446.520	1676.982	502.846
1956	390.763	74.804	52.970	110.547	612.856	334.212	309.272	490.259	1760.919	505.290
1957	395.226	75.494	55.768	113.512	627.455	340.705	315.443	516.199	1800.580	502.059
1958	391.325	75.121	58.972	121.025	640.469	347.703	322.090	541.240	1879.586	504.352
1959	400.088	76.129	60.042	128.508	662.637	355.776	329.842	568.432	1947.681	505.453
1960	408.182	76.859	59.547	136.313	676.654	364.020	333.179	586.441	1983.472	504.234
1961	403.274	77.949	61.761	144.608	689.334	367.828	337.635	609.421	2014.524	504.597
1962	406.990	79.601	63.718	147.722	686.913	372.149	353.975	640.630	2044.221	503.987
1963	405.885	81.198	63.691	157.601	707.316	373.748	379.522	666.061	2073.855	502.972
1964	412.827	81.613	62.856	164.877	736.966	376.445	413.570	694.610	2114.077	500.602
1965	414.067	86.339	64.336	172.536	753.066	380.715	456.229	724.599	2154.844	497.489
1966	409.282	89.472	63.922	178.864	772.405	381.156	487.597	753.087	2173.059	492.420
1967	418.765	94.050	63.568	184.481	778.027	379.985	506.207	769.647	2184.217	486.253
1968	418.015	89.080	58.964	194.735	795.068	380.582	529.171	787.508	2217.865	481.562
1969	424.132	92.416	55.972	188.419	822.482	384.258	545.100	815.538	2270.967	477.672

** Unadjusted: The durable X_i series were adjusted by their respective rental price normalization factors: i.e. (* p1961)

TABLE IV-8

CONSUMPTION MODEL: 'NORMALIZED' PRICES

YEAR	VF	VA	VT	VMS	VOS	VCL	VMV	VDO	VH	VL
1946	0.6959	0.9712	0.9801	0.6132	0.6778	0.7220	0.6474	0.7363	0.4961	0.2145
1947	0.6848	0.8489	0.8915	0.5565	0.6264	0.7117	0.6521	0.8253	0.4724	0.2004
1948	0.7441	0.7717	0.8307	0.5541	0.5952	0.7708	0.6891	0.7176	0.4779	0.1963
1949	0.7231	0.7350	0.8069	0.5537	0.5957	0.7662	0.6917	0.7037	0.4760	0.1950
1950	0.6787	0.6911	0.7605	0.5170	0.5825	0.7076	0.6437	0.6751	0.4622	0.2017
1951	0.6804	0.6544	0.7292	0.4932	0.5429	0.6903	0.6407	0.6664	0.4975	0.2091
1952	0.6306	0.6107	0.7002	0.4995	0.5287	0.6513	0.6086	0.6320	0.4888	0.2298
1953	0.5823	0.5852	0.5907	0.4938	0.5238	0.6187	0.5896	0.6104	0.4752	0.2246
1954	0.5565	0.5629	0.5598	0.4961	0.5197	0.5860	0.5675	0.5761	0.4643	0.3151
1955	0.5293	0.5362	0.5331	0.4851	0.5062	0.5511	0.5015	0.5510	0.4458	0.3183
1956	0.4959	0.4978	0.4939	0.4653	0.4813	0.5178	0.4754	0.5250	0.4357	0.3441
1957	0.4834	0.4798	0.4628	0.4603	0.4645	0.4892	0.4873	0.4998	0.4406	0.3822
1958	0.4807	0.4654	0.4426	0.4569	0.4584	0.4676	0.4735	0.4763	0.4215	0.4017
1959	0.4528	0.4502	0.4487	0.4469	0.4436	0.4495	0.4701	0.4535	0.4177	0.4119
1960	0.4359	0.4367	0.4382	0.4369	0.4332	0.4340	0.4528	0.4348	0.4305	0.4115
1961	0.4275	0.4275	0.4275	0.4275	0.4257	0.4275	0.4275	0.4275	0.4275	0.4275
1962	0.4242	0.4241	0.4171	0.4254	0.4247	0.4203	0.4097	0.4165	0.4153	0.4454
1963	0.4206	0.4088	0.4013	0.4180	0.4153	0.4150	0.3941	0.4040	0.4002	0.4465
1964	0.4051	0.4010	0.3864	0.4073	0.4056	0.4065	0.3606	0.3879	0.3945	0.4382
1965	0.3975	0.3849	0.3799	0.3981	0.3980	0.3936	0.3402	0.3695	0.3915	0.4120
1966	0.3957	0.3663	0.3716	0.3846	0.3882	0.3854	0.3094	0.3565	0.3996	0.4453
1967	0.3744	0.3515	0.3647	0.3858	0.3819	0.3814	0.2970	0.3526	0.4128	0.4514
1968	0.3623	0.3523	0.3819	0.3714	0.3746	0.3692	0.2823	0.3384	0.4260	0.4906
1969	0.3471	0.3399	0.3742	0.3610	0.3739	0.3569	0.2716	0.3235	0.4202	0.5201

Note: Before these v_i were calculated the rental prices were put into index form by dividing each by its respective 1961 value. The per capita real quantities for durable goods were adjusted accordingly, i.e., each x_i of a durable was multiplied by its 1961 rental price so that the final $p_i x_i$ would be unchanged.

TABLE IV-9

ANNUAL WAGE BILLS AND HOURS WORKED

YEAR	W ANNUAL WAGE BILLS ($ Millions)	AVERAGE WEEKLY HOURS
1946	6816.72656	44.88
1947	7906.87500	44.06
1948	9081.56641	43.86
1949	9772.70703	43.85
1950	10552.85547	43.00
1951	12278.62891	42.49
1952	13698.37109	42.14
1953	14872.83203	42.15
1954	15137.57031	41.97
1955	16264.78906	41.76
1956	18463.52344	41.49
1957	20060.19531	41.02
1958	20583.25391	40.68
1959	22037.83984	40.53
1960	22121.52734	40.21
1961	24197.37109	39.88
1962	25886.57812	39.57
1963	27600.56250	39.22
1964	30109.81250	38.87
1965	33663.36328	38.65
1966	38040.68359	38.32
1967	41965.41016	38,13
1968	45790.33984	37.34
1969	50796.35547	37.00

Notes:

1. This wage series is estimated to account for unincorporated business income and thus has more desirable properties than the National Accounts, which underestimate self-employed earnings, etc. The series was made available by Professors A. D. Woodland and W. E. Diewert.

2. The hours were made available through the Productivity Research Division, Statistics Canada.

TABLE IV-10

SOURCES USED TO CALCULATE
THE RATE OF TAX ON LABOUR EARNINGS

	PI = PERSONAL INCOME (Table 4 - NAT. ACCTS) ($ million) (1)	TOT. DIRECT TAX (2) (3) (4)		TABLE 44 - NAT. ACCTS) ($ million) TD	$\tau = \frac{TD}{PI}$
1946	9,887	106 - 20 - 34	=	852	.0861
1947	10,583	927 - 30 - 31	=	866	.0818
1948	12,161	986 - 29 - 29	=	928	.0763
1949	12,902	956 - 26 - 29	=	901	.0698
1950	13,681	915 - 35 - 31	=	849	.0620
1951	16,159	1279 - 35 - 34	=	1210	.0748
1952	17,900	1588 - 39 - 34	=	1515	.0846
1953	18,932	1748 - 40 - 33	=	1675	.0884
1954	19,006	1776 - 41 - 37	=	1698	.0893
1955	20,573	1855 - 55 - 72	=	1728	.0839
1956	22,817	2127 - 88 - 58	=	1981	.0869
1957	24,500	2350 - 69 - 57	=	2224	.0907
1958	25,893	2214 - 70 - 56	=	2088	.0806
1959	27,425	2444 - 76 - 54	=	2314	.0843
1960	28,921	2794 - 97 - 61	=	2636	.0911
1961	29,411	2844 - 80 - 66	=	2798	.0951
1962	31,966	3180 - 93 - 72	=	3015	.0943
1963	34,109	3387 - 89 - 82	=	3216	.0942
1964	36,618	3917 - 91 - 89	=	3737	.1020
1965	40,591	4433 - 101 - 111	=	4221	.1039
1966	45,702	5812 - 106 - 109	=	5597	.1224
1967	50,208	7011 - 96 - 119	=	6796	.1353
1968	55,213	8247 - 113 - 122	=	8012	.1451
1969	61,398	10,047 - 104 - 133	=	9810	.1597

(2) Total Direct Taxes: Table 44, National Accounts.

(3) and (4) Succession Duties and Estate Taxes: Federal and Provinces

TABLE IV-11

LABOUR: HOURS, TAX AND WAGES

YEAR	L PER CAPITA (Hours)	ℓ=H-L (Hours)	w = WGRT ($)/hr	τ = TAU	PTIME = w(1-τ) ($)/hr
1946	1180.226	1939.774	0.65	0.0861	0.59
1947	1181.111	1938.888	0.74	0.0818	0.68
1948	1169.078	1950.922	0.85	0.0763	0.78
1949	1133.498	1986.502	0.91	0.0698	0.84
1950	1110.065	2009.935	0.99	0.0620	0.92
1951	1109.850	2010.150	1.13	0.0748	1.05
1952	1086.738	2033.262	1.26	0.0846	1.15
1953	1080.266	2039.734	1.35	0.0884	1.23
1954	1052.832	2067.167	1.38	0.0893	1.25
1955	1050.747	2069.253	1.45	0.0839	1.33
1956	1067.491	2052.509	1.59	0.0868	1.46
1957	1052.953	2067.047	1.71	0.0907	1.55
1958	1016.392	2103.608	1.78	0.0806	1.63
1959	1020.807	2099.193	1.86	0.0843	1.70
1960	1011.363	2108.637	1.85	0.0911	1.68
1961	1001.436	2118.564	2.01	0.0951	1.82
1962	1002.686	2117.314	2.10	0.0943	1.91
1963	997.652	2122.348	2.21	0.0942	2.00
1964	1002.615	2117.385	2.35	0.1020	2.11
1965	1011.159	2108.841	2.54	0.1039	2.28
1966	1019.577	2100.423	2.78	0.1224	2.44
1967	1016.505	2103.495	2.99	0.1353	2.58
1968	990.483	2129.517	3.26	0.1451	2.79
1969	987.691	2132.309	3.54	0.1597	2.97

Note: In actual processing, PTIME was put in index form $\left(\frac{PTIME_t}{PTIME_{1961}}\right)$ and H-L was adjusted accordingly, i.e., $(H\text{-}L)_t$* PTIME 1961.

TABLE IV-12

CONSUMPTION AND LEISURE: 'NORMALIZED' PRICES

YEAR	VF	VA	VT	VMS	VOS	VCL	VMV	VDO	VH	VL	VTIME
1946	0.2906	0.4055	0.4092	0.2560	0.2830	0.3015	0.2703	0.3074	0.2071	0.0895	0.1654
1947	0.2866	0.3553	0.3731	0.2329	0.2621	0.2979	0.2729	0.3454	0.1977	0.0839	0.1652
1948	0.3067	0.3181	0.3424	0.2283	0.2453	0.3177	0.2840	0.2957	0.1970	0.0809	0.1660
1949	0.2904	0.2952	0.3241	0.2224	0.2393	0.3077	0.2778	0.2826	0.1912	0.0783	0.1660
1950	0.2692	0.2741	0.3017	0.2051	0.2311	0.2807	0.2554	0.2678	0.1834	0.0800	0.1654
1951	0.2717	0.2613	0.2911	0.1969	0.2167	0.2756	0.2558	0.2661	0.1986	0.0835	0.1647
1952	0.2461	0.2383	0.2733	0.1949	0.2063	0.2542	0.2375	0.2467	0.1907	0.0897	0.1652
1953	0.2242	0.2253	0.2274	0.1901	0.2016	0.2382	0.2270	0.2350	0.1829	0.0864	0.1661
1954	0.2153	0.2178	0.2166	0.1919	0.2011	0.2267	0.2196	0.2229	0.1796	0.1219	0.1634
1955	0.2034	0.2060	0.2049	0.1864	0.1945	0.2118	0.1927	0.2117	0.1713	0.1223	0.1639
1956	0.1900	0.1907	0.1893	0.1783	0.1844	0.1984	0.1822	0.2012	0.1670	0.1319	0.1656
1957	0.1844	0.1830	0.1765	0.1756	0.1772	0.1866	0.1859	0.1907	0.1681	0.1458	0.1649
1958	0.1807	0.1750	0.1664	0.1718	0.1723	0.1758	0.1780	0.1791	0.1585	0.1510	0.1634
1959	0.1717	0.1708	0.1702	0.1695	0.1682	0.1705	0.1783	0.1720	0.1584	0.1562	0.1629
1960	0.1704	0.1707	0.1713	0.1708	0.1693	0.1697	0.1770	0.1700	0.1683	0.1608	0.1592
1961	0.1617	0.1617	0.1617	0.1617	0.1617	0.1617	0.1617	0.1617	0.1617	0.1617	0.1617
1962	0.1585	0.1585	0.1559	0.1590	0.1587	0.1570	0.1531	0.1556	0.1552	0.1664	0.1630
1963	0.1558	0.1515	0.1487	0.1549	0.1539	0.1537	0.1460	0.1497	0.1483	0.1654	0.1634
1964	0.1498	0.1483	0.1429	0.1507	0.1500	0.1504	0.1334	0.1435	0.1459	0.1621	0.1640
1965	0.1448	0.1402	0.1384	0.1450	0.1450	0.1434	0.1239	0.1346	0.1426	0.1501	0.1661
1966	0.1440	0.1333	0.1353	0.1400	0.1413	0.1403	0.1126	0.1298	0.1455	0.1621	0.1668
1967	0.1362	0.1278	0.1326	0.1403	0.1389	0.1387	0.1080	0.1282	0.1501	0.1642	0.1667
1968	0.1301	0.1265	0.1371	0.1334	0.1345	0.1326	0.1014	0.1215	0.1530	0.1762	0.1658
1969	0.1250	0.1224	0.1347	0.1300	0.1346	0.1285	0.0978	0.1165	0.1513	0.1873	0.1653

```
C   PROGRAM TO CALCULATE A DEPRECIATED STOCK SERIES
C   ASSUME CARRYOVER STOCK DEPRECIATES AT SAME INSTANT NEW INVEST-
C   MENT OCCURS.
       DIMENSION STOCK(30),SPEND(30),IND(30),ITITLE(3)
3      READ(5,1,END=99) NYEAR,IBASE,ITITLE
1      FORMAT(2I5,3A4)
       READ(5,2) DELTA,RINIT,(SPEND(I),I=1,NYEAR)
2      FORMAT(F3.2,F10.0,/(8F10.0))
       STOCK(1)=(RINIT*(1.-DELTA)) + SPEND(1)
       DO 10 I=2,NYEAR
       STOCK(I)=(STOCK(I-1))*(1.-DELTA) +SPEND(I)
10     CONTINUE
       WRITE(6,35)
 35    FORMAT(///)
       WRITE(6,11) ITITLE,DELTA,IBASE,RINIT
11     FORMAT(15X,'STOCK SERIES FOR',1X,3A4/15X,'DEPRECIATED AT',1X,F3.2,
      1'  YEARLY.'/15X,'BASE YEAR IS',1X,I4/15X,'INITIAL STOCK =',2X,F10.
      24/15X,'($MILLIONS)'//)
       WRITE(6,80)
80     FORMAT(18X,'YEAR',4X,'PURCHASES',10X,'STOCK'/18X,32('='))
       IND(1)=IBASE
       DO 23 L=2,NYEAR
 23    IND(L) = IND(L-1) +1
       DO 50 K=1,NYEAR
50     WRITE(6,12)IND(K),SPEND(K),STOCK(K)
12     FORMAT(18X,I4,3X,F10.4,F15.5)
       GO TO 3
 99    STOP
       END
```

PROGRAM DELTA

```
C    PROGRAM TO CALCULATE THE TERMS OF THE GEN.LEONTIEF.FUNCTIONAL FORM
C   DENOTE SUM OVER INDEX 'I' BY    E/I=1,N/
C   AND THE KRONECKER DELTA BY       D/I,J/
C
C   UPON MAKING THE NORMALIZATION
C
C       1 = (2./SQRT(V*)E/N=1,N/ B0N + E/I=1,N/E/J=1,N/ B(I,J)
C    AND SOLVING FOR B(1,1)    (WHERE V* IS THE COMMON PRICE IN 1961)
C
C   B(1,1)= 1 - (2/SQRT(V*)E/N=1,N/ B0(N) - 2*EE/1<|=I<J<|=N/ B(I,J)
C   - E/I=2,N/ B(I,I)
C
C   WE FIND THAT THE SYSTEM OF CONSUMER DEMAND EQUATIONS IS GIVEN BY:
C
C
C   V(I)X(I)/I=2,N/ =
C   (1)       E/N=1,N/ B0(N)*(V(I)X(I)-SQRT(V(N))V(I)X(I)/V(1)
C   +  D/I,N/ SQ(VI)/V(1))
C
C   (2)(A)   +EE/1<|=K<J<|=N/ B(K,J)*(2V(I)X(I)-2SQ(V(K)V(J))V(I)X(I)/V(1)
C       (B)   +D/I,K/SQRT(V(I)V(J))/V(1)
C       (C)   +D/I,J/SQRT(V(K)V(I))/V(1)
C
C   (3)       +E/L=2,N/ B(L,L)*(V(I)X(I) - V(L)V(I)X(I)/V(1)
C   + D/I,L/ V(I)/V(1))
C
C   NYEAR = NO. OF OBSERVATIONS
C   N=# OF VARIABLES
C   A(I,J,K,L) = B(I,J) FOR THE KTH EQUATION AND THE LTH YEAR ( OBS.)
C     FOR K=2,N
C   A(I,K,1,L) = B0(I) FOR THE KTH EQUATION AND THE LTH YEAR
C       DIMENSION A(20,20,20,25), SV(20),V(20),X(20)
        EXTERNAL LEONS
        COMMON SV(20),V(20),X(20),NYEAR,N,BASE
        READ(5,1) NYEAR
 1      FORMAT(I5)
        READ(5,2) N
 2      FORMAT(I5)
        READ(5,163) BASE
 163    FORMAT(F7.0)
        WRITE(6,1002) BASE
1002    FORMAT(1X,F7.3)
C   DYNAMIC STORAGE ALLOCATION REPLACES DIMENSION
C   STATEMENT FOR ARRAY A IN MAIN PROGRAM
        IS=NYEAR*N*N*N*4
        CALL GSPACE(A,IS)
        CALL CALLER(LEONS,A)
        STOP
        END
        SUBROUTINE LEONS(A)
        DIMENSION A(N,N,N,NYEAR)
        COMMON SV(20),V(20),X(20),NYEAR,N,BASE
        DO 55 LY=1,NYEAR
        READ(8,3) (V(I),I=1,N)
 3      FORMAT(5X,15F5.0)
        DO 5 I=1,N
 5      SV(I)=SQRT(V(I))
        READ(8,19) (X(I),I=1,N)
 19     FORMAT(12F9.6)
        N1=N-1
```

```
      DO 50 I=2,N
      VX=V(I)*X(I)
      VF=VX/V(1)
      DO 10 L=2,N
C   **CALC (3)
10    A(L,L,I,LY) =VX-V(L)*VF
      A(I,I,I,LY) = A(I,I,I,LY) +V(I)/V(1)
      DO 20 K=1,N1
C   **CALC (2)(A)   THE UPPER TRIANGLE OF B(I,J)
      VFK=VF*SV(K)
      K1=K+1
      DO 21 J=K1,N
21    A(K,J,I,LY) =2.*(VX -SV(J)*VFK)
20    CONTINUE
      SVV=SV(I)/V(1)
      IF(I.EQ.N) GO TO 51
C   **CALC (2)(B)    FOR K=I   D/I,K/
      K1=I+1
      DO 31 J=K1,N
31    A(I,J,I,LY)=A(I,J,I,LY) + SV(J)*SVV
C   **CALC (2)(C)    FOR J=I   D/I,J/
51    J1=I-1
      DO 32 KK=1,J1
C   **CALC (1)         THE BJ(I) TERMS
32    A(KK,I,I,LY) = A(KK,I,I,LY) + SV(KK)*SVV
      DO 40 NN=1,N
40    A(NN,I,1,LY) = (2./BASE)*VX-SV(NN)*VF
      A(I,I,1,LY) = A(I,I,1,LY) + SV(I)/V(1)
50    CONTINUE
55    CONTINUE
C    DATA ARE OUTPUTTED UNFORMATTED ; MUST READ IN FREE FORMAT
      WRITE(7) (((A(I,J,1,L),L=1,NYEAR),J=2,N),I=1,N)
      DO 56 I=1,N
      I1=I
C   CHECK FOR NO BB11 TERM
      IF(I.EQ.1) I1=2
      WRITE(7) (((A(I,J,K,L),L=1,NYEAR),K=2,N),J=I1,N)
56    CONTINUE
      STOP
      END
```

PROGRAM LF

```
C  PROGRAM TO CALCULATE ELASTICITIES OF SUBSTITUTION FOR THE GENERALIZED
C  LEONTIEF UTILITY FUNCTION    (UNNORMALIZED DEMANDS)
C   INPUT N= NUMBER OF GOODS  (B COEFFICIENTS ARE IN AN N BY N ARRAY
C  WHICH IS SYMMETRIC . I.E.  B(I,J) = B(J,I)......
      COMMON B,B1,SV,N
      DIMENSION V(30),SV(30),B(30,30),B1(30),HIA(30)
      READ(5,8)N
8     FORMAT(I5)
      DO 6 I=1,N
6     READ(5,4)(B(I,J),J=1,N)
4     FORMAT(F11.8)
      READ(5,4)(B1(I),I=1,N)
  57  READ(5,3,END=888)IYEAR,(V(I),I=1,N)
 3    FORMAT(I4,1X,(15F5.0))
      WRITE(6,718)IYEAR
 718  FORMAT(14X,I4///)
      DO 10 I=1,N
10    SV(I)=SQRT(V(I))
      DO 16 I=1,N
      HIA(I)=HI(I)
16    CONTINUE
      DO 993 I=1,N
      WRITE(6,113) I,HIA(I)
  113 FORMAT(1X,'H',I3,4X,F12.5)
      DO 993 J=I,N
      IF(J.EQ.I) GO TO 996
 13   PDER   =HIJ(I,J)
      WRITE(6,114)I,J,PDER
 114  FORMAT(1X,'H',I2,I2,3X,F12.5)
      GO TO 993
 996  PDER   =HII(I)
 992  WRITE(6,114)I,J,PDER
  993 CONTINUE
      Y=0.
      UTIL=0.
      DO 54 I=1,N
      S=0.
      Y=Y+B1(I)*SV(I)
      DO 53 J=1,N
53    S=S+B(I,J)*SV(J)
54    UTIL=UTIL+S*SV(I)
      UTIL=UTIL+2.*Y
      DO111 I=1,N
      DO 111 J=I,N
      IF(I.EQ.J) GO TO 49
      SIGMA=UTIL*HIJ(I,J)/(HIA(I)*HIA(J))
      GO TO 111
49    X=HIA(I)
      SIGMA=(UTIL*HII(I))/(X*X)
111   WRITE(6,30) I,J,SIGMA
30    FORMAT(10X,'SIGMA',I2,I2,2X,'=',F12.6)
      GO TO 57
888   STOP
      END
      FUNCTION HI(I)
      COMMON B,B1,SV,N
      DIMENSION B(30,30),B1(30),SV(30)
      X=0.
      DO 10 K=1,N
```

```
10    X=X+B(I,K) *(SV(K)/SV(I))
      HI=X + B1(I)/SV(I)
      RETURN
      END
      FUNCTION HIJ(I,J)
      COMMON B,B1,SV,N
      DIMENSION B(30,30),B1(30),SV(30)
      HIJ= .5 * B(I,J)/(SV(I)*SV(J))
      RETURN
      END
      FUNCTION HII(I)
      COMMON B,B1,SV,N
      DIMENSION B(30,30),B1(30),SV(30)
      X=0.
      DO 10 J=1,N
      IF(J.EQ.I) GO TO 10
      X=X+B(I,J) *(SV(J)/SV(I)**3)
10    CONTINUE
      HII=-.5*(X+B1(I)/SV(I)**3)
      RETURN
      END
```

PROGRAM SIGUNN

```
C   PRCGRAM TC CALCULATE ELASTICITIES CF SUBSTITUTION FCR THE GENERALIZ
C   LECNTIEF UTILITY FUNCTICN      (NCRMALIZEC DEMANDS)
C    INPUT N= NUMBER OF GOCDS  (B COEFFICIENTS ARE IN AN N BY N ARRAY
C   WHICH IS SYMMETRIC . I.E.  B(I,J) = B(J,I)......
      CCMMCN B,B1,SV,N
      DIMENSION V(30),SV(3C),B(3C,30),B1(3C),HIA(30),HIJA(30,30)
      READ(5,8)N
8     FCRMAT(I5)
      DC 6 I=1,N
6     READ(5,4)(B(I,J),J=1,N)
4     FCRMAT(F11.8)
      READ(5,4)(B1(I),I=1,N)
  57  READ(5,3,END=888)IYEAR,(V(I),I=1,N)
 3    FCRMAT(I4,1X,(15F5.C))
      WRITE(6,718)IYEAR
 718  FCRMAT(14X,I4///)
      DC 10 I=1,N
1C    SV(I)=SCRT(V(I))
      DC 16 I=1,N
      HIA(I)=HI(I)
      DC 16 J=1,N
      IF(I.EC.J) GC TO 705
      HIJA(I,J)=HIJ(I,J)
      GC TC 16
  7C5 HIJA(I,J)=HII(I)
16    CCNTINUE
      DO 993 I=1,N
      WRITE(6,113) I,HIA(I)
  113 FCRMAT(1X,'H',I3,4X,F12.5)
      DC 993 J=I,N
      IF(J.EC.I) GC TO 996
  13  WRITE(6,114)I,J,HIJA(I,J)
 114  FCRMAT(1X,'H',I2,I2,3X,F12.5)
      GO TO 993
 996  PDER   =HII(I)
 992  WRITE(6,114)I,J,PDER
  993 CONTINUE
      SCN=C.0
      SCD=C.C
      DC 91 K=1,N
      SCD=SCD+(V(K)*HIA(K))
      DC 91 M=1,N
91    SCN=SCN+(V(K)*HIJA(K,M)*V(M))
      SC=SCN/SDD
      DC111 I=1,N
      WRITE(6,393) I,V(I)
 393  FCRMAT(4X,'V(',I2,') =',4X,F9.5)
      DC 111 J=I,N
      SA=C.C
      SB=C.C
      SC=0.0
      CC=HIJA(I,J)/(HIA(I)*HIA(J))
      DC 48 K=1,N
      SA=SA+V(K)*HIA(K)
      SB=SB+(V(K)*HIJA(J,K))
      SC=SC+(V(K)*HIJA(I,K))
48    CCNTINUE
      SA=SA*CC
      SB=SB/HIA(J)
```

```
      SC=SC/HIA(I)
      SIGMA=SA-SB-SC+SD
111   WRITE(6,30) I,J,SIGMA
30    FORMAT(10X,'(NORMALIZED DEMAND) SIGMA',1X,I2,I2,2X,'=',F12.6)
      WRITE(6,2) IYEAR
  2   FORMAT(12X,//12X,'ESTIMATED MARKET SHARES',4X,I4/)
      DO 24 I=1,N
      SHARE=0.0
      SUM=0.0
      DO 25 K=1,N
 25   SUM=SUM + V(K)*HIA(K)
      SHARE=V(I)*(HIA(I)/SUM)
  24  WRITE(6,65) I,SHARE
 65   FORMAT(18X,'GOOD',1X,I2,1X,':',4X,F14.6)
      GO TO 57
888   STOP
      END
      FUNCTION HI(I)
      COMMON B,B1,SV,N
      DIMENSION B(30,30),B1(30),SV(30)
      X=0.
      DO 10 K=1,N
10    X=X+B(I,K) * SV(K)
      HI=(X + B1(I))/SV(I)
      RETURN
      END
      FUNCTION HIJ(I,J)
      COMMON B,B1,SV,N
      DIMENSION B(30,30),B1(30),SV(30)
      HIJ= .5 * B(I,J)/(SV(I)*SV(J))
      RETURN
      END
      FUNCTION HII(I)
      COMMON B,B1,SV,N
      DIMENSION B(30,30),B1(30),SV(30)
      X=0.
      DO 10 J=1,N
      IF(J.EQ.I) GO TO 10
      X=X+ B(I,J) *SV(J)
10    CONTINUE
      HII=-.5*(X+B1(I))/(SV(I)**3)
      RETURN
      END
```

PROGRAM SIGNORM

```
C  PROGRAM TC CALCLLATE THE SHARES EACH GCCC IS ESTIMATEC TO
C HAVE CF THE MARKET (I.E. THE ESTIMATED V(I)X(I) GIVEN THE
C  COEFFICIENTS ESTIMATED BY THE GEN. LECNTIEF FUNCTICN
      DIMENSICN V(3C),SV(3C),B(30,3C),B1(3C),BNCT(3C)
      READ(5,8)N
8     FCRMAT(I5)
      DC 6 I=1,N
6     READ(5,4)(B(I,J),J=1,N)
4     FCRMAT(F11.8)
      READ(5,4)(B1(I),I=1,N)
  57  READ(5,3,END=888)IYEAR,(V(I),I=1,N)
 3    FCRMAT(I4,1X,(15F5.C))
      WRITE(8,718)IYEAR
 718  FCRMAT(14X,I4///)
      DC 10 I=1,N
 1C   SV(I)=SGRT(V(I))
      DEN1=C.
      DEN2=0.
      DC 5 K=1,N
      DEN2=DEN2 + B1(K)*SV(K)
      DC 5 J=1,N
      DEN1=DEN1+ B(K,J)*SV(K)*SV(J)
  5   CCNTINUE
      DEN=DEN1 + DEN2
      DO 11 L=1,N
  11  BNCT(L)=B1(L)/SV(L)
      DC 2 I=2,N
      RN1=0.
      DO 9 J=1,N
  9   RN1=RN1 +B(I,J)*SV(J)/SV(I)
      SHARE=V(I)*(RN1 + BNCT(I))/DEN
      WRITE(6,7) I,SHARE
  7   FCRMAT(1X,'MARKET SHARE, GOCD',1X,I2,1X,'=',2X,F1C.4)
  2   CCNTINUE
      GC TC 57
 888  STOP
      ENC
```

PROGRAM MSHARE

CHAPTER V

REGRESSION RESULTS
MODEL NO. 1 CONSUMPTION ONLY

Note: To make notation more concise, the goods will be referred to by the following subscripts:

FOOD	-	1
ALCOHOL	-	2
TOBACCO	-	3
MEDICAL SERVICES	-	4
OTHER SERVICES	-	5
CLOTHING	-	6
MOTOR VEHICLES	-	7
OTHER DURABLES	-	8
HOUSING	-	9
LAND	-	10

TABLE V-1

ESTIMATED (2nd stage) COEFFICIENTS FOR CONSUMER PREFERENCES:

Consumption Only

COEFFICIENT	ESTIMATE	(t-statistic)	STANDARD ERROR
B12	-.0025	(-0.7823)	.0035
B13	-.0063	(-2.9355)	.0021
B14	-.0180	(-4.6343)	.0039
B15	-.0269	(-3.7712)	.0071
B16	-.0119	(-2.3599)	.0050
B22	-.0135	(-3.7758)	.0036
B23	.0026	(1.3061)	.0020
B24	.0136	(3.3436)	.0041
B25	-.0005	(-0.0949)	.0054
B26	-.0051	(-1.6764)	.0030
B27	-.0049	(-3.5594)	.0014
B28	.0044	(3.2139)	.0014
B33	-.0026	(-1.5607)	.0016
B34	.0084	(2.9886)	.0028
B35	-.0106	(-3.1528)	.0034
B36	-.0007	(-0.3410)	.0019
B44	-.0201	(-2.4041)	.0083
B45	-.0540	(-6.0065)	.0090
B55	-.0078	(-0.5078)	.0154
B56	-.0186	(-3.3756)	.0055
B57	-.0161	(-5.3816)	.0030
B58	-.0290	(-7.2237)	.0040
B66	.0085	(1.7189)	.0050
B67	.0035	(2.4861)	.0014
B68	-.0047	(-2.9154)	.0016
B69	-.0282	(-11.1185)	.0025
B77	-.0101	(-7.4168)	.0014
B78	-.0094	(-7.7906)	.0012
B79	-.0143	(-6.4622)	.0022
B7,10	-.0016	(-2.6079)	.0006
B88	-.0161	(-7.9989)	.0020

TABLE V-1 (Continued)

COEFFICIENT	ESTIMATE	(t-statistic)	STANDARD ERROR
B89	-.0345	(-18.1241)	.0019
B8, 10	-.0046	(- 5.7505)	.0008
B99	-.0005	(- 0.0788)	.0061
B9,10	-.0062	(- 4.9008)	.0013
B10,10	.0070	(10.8453)	.0006
Bo1	.0716	(23.5036)	.0030
Bo2	.0080	(4.3452)	.0019
Bo3	.0091	(9.1172)	.0010
Bo4	.0538	(23.8655)	.0023
Bo5	.1439	(38.8561)	.0037
Bo6	.0473	(21.9156)	.0022
Bo7	.0417	(21.4250)	.0019
Bo8	.0723	(42.2632)	.0017
Bo9	.0736	(20.6606)	.0036
Bo10	.0075	(6.8246)	.0011

By substituting, we solve for

$$b_{11} = -.01121$$

Summary Statistics: Consumption Only

$$R^2 = 1.0000$$

Standard Error of the Regression = 0.99827

Number of Observations = 216

If an estimated b_{ij} is positive, the goods i and j are, in a sense, pseudo-substitutes. True substitution and complementarity are estimated and tabled in Table V-3 (elasticities of substitution).

We may use the estimated coefficients to calculate the partial derivatives of the indirect utility function*, and from these partials and our 'normalized' prices, we may predict market shares. The predicted shares are reported and compared with actual shares in Table V-2 for the first and last years in our analysis.

TABLE V-2

MODEL NO. 1 FITTED MARKET SHARES

(YEAR 1) 1946 GOOD	ACTUAL $= v_i x_i$	FITTED $= v_i h_i / \Sigma_k v_k h_k$	where $h_i = \frac{\partial h(v)}{\partial v_i}$
(1) FOOD	.2601	.2697	
(2) ALCOHOL	.0518	.0471	
(3) TOBACCO	.0449	.0547	
(4) MEDICAL SERVICES	.0473	.0144	
(5) OTHER SERVICES	.3074	.2989	
(6) CLOTHING	.1013	.0995	
(7) MOTOR VEHICLES	.0090	−.0128	thus σ_{i7} will not be defined
(8) OTHER DURABLES	.0465	.0396	
(9) HOUSING	.1173	.1657	
(10) LAND	.0146	.0232	
(YEAR 24) 1969			
(1) FOOD	.1474	.1547	
(2) ALCOHOL	.0314	.0310	
(3) TOBACCO	.0210	.0226	
(4) MEDICAL SERVICES	.0680	.0700	
(5) OTHER SERVICES	.3075	.3047	
(6) CLOTHING	.0714	.0715	
(7) MOTOR VEHICLES	.0560	.0544	
(8) OTHER DURABLES	.0896	.0864	
(9) HOUSING	.1703	.1670	
(10) LAND	.0374	.0378	

*i.e., derived demands.

Corresponding to each non-zero off-diagonal b_{ij}, we may calculate a Hicks-Allen (partial) elasticity of substitution (σ_{ij}) between the demands i and j, in a particular time period (i.e., year) and evaluated at the prevailing prices. While we have restricted certain b_{ij} to be zero *a priori*, this does not imply that $\sigma_{ij} = 0$ (whereas $\hat{\sigma}_{ij}$, the pseudo elasticity, will be = 0), since there are *general* substitution effects in our normalized demand system (see Chapter III, equation III-7). If σ_{ii} is negative we then know that the elasticities of demand are of the correct sign. If, however, we estimate a negative share for a good, the related σ_{ij} will be meaningless.

TABLE V-3

σ_{ij}: ELASTICITIES OF SUBSTITUTION
MODEL NO. 1
CONSUMPTION ONLY

(For Normalized Market Demands)

$\sigma_{ij} > 0 \rightarrow$ i,j SUBSTITUTES

$\sigma_{ij} < 0 \rightarrow$ i,j COMPLEMENTS

	1946	1961	1969
σ1 1	-3.925	-2.688	-2.892
σ1 2	-4.496	-0.628	-0.380
σ1 3	-8.738	-2.748	-2.290
σ1 4	-6.675	-1.354	-0.876
σ1 5	-0.884	0.113	0.202
σ1 6	-2.608	-0.347	-0.142
σ1 7	-62.702	2.475	1.840
σ1 8	30.344	2.845	2.235
σ1 9	2.356	1.505	1.249
σ1 10	-1.288	0.502	0.510

TABLE V-3 (Continued)

	1946	1961	1969
σ 2 2	-144.113	-45.132	-40.963
σ 2 3	20.161	8.159	7.392
σ 2 4	404.530	18.377	12.950
σ 2 5	2.378	0.636	0.538
σ 2 6	- 16.990	- 3.664	- 2.943
σ 2 7	77.774	- 4.778	- 3.333
σ 2 8	73.658	5.952	4.432
σ 2 9	1.168	0.776	0.621
σ 2 10	- 2.475	- 0.226	- 0.117
σ 3 3	- 30.006	-29.018	-30.926
σ 3 4	242.275	15.898	12.086
σ 3 5	- 8.820	- 2.405	- 1.851
σ 3 6	0.897	0.843	0.922
σ 3 7	- 63.952	2.233	1.709
σ 3 8	29.095	2.603	2.104
σ 3 9	1.107	1.262	1.118
σ 3 10	- 2.537	0.259	0.379
σ 4 4	-1231.802	--15.813	-11.654
σ 4 5	- 115.493	- 4.222	- 2.569
σ 4 6	64.579	3.588	2.798
σ 4 7	- 2.518	4.145	2.820
σ 4 8	90.530	4.514	3.215
σ 4 9	62.541	3.174	2.229
σ 4 10	58.897	2.171	1.490
σ 5 5	5.134	-0.149	- 0.453
σ 5 6	- 1.473	0.138	0.284
σ 5 7	2.849	0.265	0.324
σ 5 8	- 4.411	0.204	0.333
σ 5 9	6.260	1.777	1.334
σ 5 10	2.616	0.775	0.595
σ 6 6	17.303	-0.152	- 1.456
σ 6 7	-100.767	5.196	3.946
σ 6 8	15.199	1.810	1.545
σ 6 9	- 16.422	-3.914	- 2.859
σ 6 10	2.787	1.196	1.146

TABLE V-3 (Continued)

	1946	1961	1969
σ 7 7	-969.432	-13.159	-11.523
σ 7 8	250.097	- 0.780	- 0.165
σ 7 9	24.175	- 1.593	- 0.853
σ 7 10	- 18.575	- 0.634	- 0.391
σ 8 8	-121.479	- 6.678	- 6.117
σ 8 9	- 38.431	- 3.668	- 2.257
σ 8 10	- 15.213	- 2.171	- 1.403
σ 9 9	1.181	- 1.582	- 1.715
σ 9 10	- 10.930	- 2.609	- 1.794
σ 10 10	37.368	2.234	- 0.236

Discussion

The second-stage equation for consumer expenditures alone is an excellent fit insofar as predicting market shares is concerned, and an F test reveals that we must reject the hypotheses that

(i) $b_{oj} = 0 \; \forall_j$ (i.e., homothetic utility function); and

(ii) $b_{ij} = 0 \; \forall_i \neq j$ (i.e., [fixed proportion] Leontief preferences)

Consumption Model

HYPOTHESES

H_o:

(i) $b_{oj} = 0 \; \forall_j$ (i.e., homothetic utility function)

$F(10, 170) = 3673.107$

Thus, reject H_o.

(ii) H_o:

$$b_{ij} = 0 \ \forall_i \neq j \quad \text{(i.e., Leontief preferences)}$$

$F(27, 170) = 162.181$

Thus, reject H_o.

In Chapter VI we examine the results for the consumer demand system when the demand for leisure is simultaneously determined.

CHAPTER VI

REGRESSION RESULTS
MODEL NO. 2 CONSUMPTION AND LEISURE

The notation in this chapter is identical to that of Chapter V, save for the addition of leisure, which will be denoted by subscript 11 (eleven).

TABLE VI-1

ESTIMATED (2nd stage) COEFFICIENTS FOR CONSUMER PREFERENCES: CONSUMPTION AND LEISURE

COEFFICIENT	ESTIMATE	(t-statistic)	STANDARD ERROR
B12	-.0019	(-3.1237)	.0006
B13	-.0005	(-1.6551)	.0003
B14	-.0014	(-4.5962)	.0003
B15	-.0027	(-2.6453)	.0010
B16	-.0030	(-3.9616)	.0008
B1,11	-.0269	(-7.8502)	.0034
B22	-.0022	(-3.3116)	.0007
B23	.0004	(1.3777)	.0003
B24	.0009	(1.9234)	.0004
B25	.0004	(0.3838)	.0010
B26	-.0011	(-2.1253)	.0005
B27	.0012	(3.8618)	.0003
B28	.0007	(2.6305)	.0003
B2,11	.0047	(2.4284)	.0019
B33	-.0005	(-2.5047)	.0002
B34	.0005	(2.1353)	.0002
B35	-.0020	(-4.7626)	.0004
B36	-.0001	(-0.5266)	.0003
B37	.0003	(1.2207)	.0002
B3,11	-.0086	(-7.3576)	.0012

TABLE VI-1 (Continued)

COEFFICIENT	ESTIMATE	(t-statistic)	STANDARD ERROR
B44	-.0008	(-1.4507)	.0006
B45	-.0047	(-6.6332)	.0007
B4,11	-.0028	(-2.3628)	.0012
B55	-.0063	(-3.0447)	.0021
B56	-.0007	(-0.7844)	.0009
B57	-.0026	(-4.8998)	.0005
B58	-.0018	(-3.2965)	.0005
B5,11	-.0465	(-12.3860)	.0038
B66	.0005	(0.6035)	.0008
B67	.0010	(3.1338)	.0003
B68	-.0009	(-3.0344)	.0003
B69	-.0031	(-7.4567)	.0004
B6,11	-.0094	(-3.3364)	.0028
B77	-.0022	(-7.5456)	.0003
B78	-.0010	(-4.0444)	.0002
B79	-.0032	(-10.3469)	.0003
B7,10	-.0011	(-6.3310)	.0002
B7,11	-.0176	(-8.3547)	.0021
B88	-.0018	(-7.7498)	.0002
B89	-.0031	(-10.6450)	.0003
B8,10	-.0009	(-6.4149)	.0001
B8,11	-.0089	(-4.6242)	.0019
B99	-.0028	(-3.5914)	.0008
B9,10	-.0028	(-11.7311)	.0002
B9,11	-.0438	(-15.4344)	.0028
B10,10	-.0017	(-9.5607)	.0002
B10,11	-.0255	(-16.4287)	.0016
B11,11	-.4874	(-39.1376)	.0125
Bo1	.0164	(10.4217)	.0016
Bo2	-.0011	(- 1.3065)	.0008
Bo3	.0044	(8.8181)	.0005

TABLE VI-1 (Continued)

COEFFICIENT	ESTIMATE	(t-statistic)	STANDARD ERROR
Bo4	.0036	(6.2185)	.0006
Bo5	.0281	(15.7032)	.0018
Bo6	.0071	(5.5029)	.0013
Bo7	.0103	(11.3666)	.0009
Bo8	.0075	(8.1338)	.0009
Bo9	.0243	(17.6500)	.0014
Bo10	.0130	(18.4353)	.0007
Bo11	.2775	(82.0368)	.0034

By substituting, we solve for

$$b_{11} = -.002257$$

Summary Statistics: Consumption and Leisure

$R^2 = 1.000$

Standard Error of the Regression = 0.8422

Number of Observations = 240

The predicted market shares are more accurate (i.e., close to actual $v_i x_i$) than those of Model No. 1, especially in the later years.

TABLE VI-2

MODEL NO. 2
CONSUMPTION AND LEISURE

FITTED MARKET SHARES

(YEAR 1) 1946 GOOD	ACTUAL = $v_i x_i$	FITTED = $v_i h_i / \Sigma_k v_k h_k$
(1) FOOD	.1092	.0920
(2) ALCOHOL	.0216	.0248
(3) TOBACCO	.0187	.0172
(4) MEDICAL SERVICES	.0197	.0222
(5) OTHER SERVICES	.1283	.1244
(6) CLOTHING	.0422	.0455
(7) MOTOR VEHICLES	.0036	.0116
(8) OTHER DURABLES	.0194	.0266
(9) HOUSING	.0489	.0436
(10) LAND	.0061	.0082
(11) LEISURE	.5823	.5838
(YEAR 24) 1969		
(1) FOOD	.0534	.0598
(2) ALCOHOL	.0113	.0108
(3) TOBACCO	.0075	.0079
(4) MEDICAL SERVICES	.0245	.0248
(5) OTHER SERVICES	.1107	.1113
(6) CLOTHING	.0257	.0272
(7) MOTOR VEHICLES	.0201	.0185
(8) OTHER DURABLES	.0323	.0304
(9) HOUSING	.0614	.0598
(10) LAND	.0134	.0120
(11) LEISURE	.6397	.6374

TABLE VI-3

σ_{ij}: ELASTICITIES OF SUBSTITUTION

MODEL NO. 2
CONSUMPTION AND LEISURE

(For Normalized Market Demands)

	1946	1961	1969
σ 1 1	- 1.070	-13.903	-12.291
σ 1 2	25.516	-53.099	-34.719
σ 1 3	1.941	- 2.356	- 0.380
σ 1 4	11.770	-16.304	- 9.606
σ 1 5	1.336	- 1.418	- 1.363
σ 1 6	13.234	-24.493	-15.119
σ 1 7	-26.063	17.087	8.994
σ 1 8	- 5.530	4.981	1.982
σ 1 9	-12.020	13.076	7.669
σ 1 10	-26.758	36.544	31.802
σ 1 11	- 0.692	1.668	1.333
σ 2 2	93.377	-326.788	-244.670
σ 2 3	-24.424	49.719	49.018
σ 2 4	-25.888	39.822	20.477
σ 2 5	2.219	- 4.806	- 4.256
σ 2 6	29.728	- 64.609	- 42.821
σ 2 7	-107.904	85.525	48.361
σ 2 8	- 22.928	23.122	11.923
σ 2 9	- 3.727	- 2.675	- 2.514
σ 2 10	- 18.464	20.793	21.619
σ 2 11	- 5.350	9.199	6.426
σ 3 3	4.295	- 94.013	-120.905
σ 3 4	- 34.266	42.907	34.122
σ 3 5	13.784	- 18.164	- 12.824
σ 3 6	- 0.893	2.695	3.919
σ 3 7	- 58.734	42.842	32.224
σ 3 8	- 9.435	10.648	9.979
σ 3 9	- 16.435	18.742	15.666
σ 3 10	- 31.172	42.211	39.799
σ 3 11	2.327	- 2.144	- 2.161

TABLE VI-3 (Continued)

	1946	1961	1969
σ 4 4	6.682	- 53.936	- 37.497
σ 4 5	28.529	- 33.141	- 18.142
σ 4 6	0.024	0.480	- 0.666
σ 4 7	-25.206	12.518	5.632
σ 4 8	- 4.674	0.412	- 1.380
σ 4 9	-11.164	8.506	4.306
σ 4 10	-25.901	31.975	28.439
σ 4 11	- 4.166	5.892	3.749
σ 5 5	- 1.065	- 8.636	- 7.433
σ 5 6	0.087	0.706	- 0.074
σ 5 7	5.534	- 4.157	- 2.395
σ 5 8	3.309	- 4.592	- 3.326
σ 5 9	-13.649	12.951	7.363
σ 5 10	-28.386	36.419	31.496
σ 5 11	- 0.411	1.617	1.314
σ 6 6	-15.850	- 2.361	- 9.425
σ 6 7	-62.220	47.795	26.227
σ 6 8	10.384	- 14.080	- 8.579
σ 6 9	14.302	- 20.716	-12.685
σ 6 10	-26.059	35.815	31.536
σ 6 11	- 2.043	3.924	2.784
σ 7 7	195.399	- 91.349	-59.615
σ 7 8	32.976	- 13.011	- 6.762
σ 7 9	62.871	- 25.108	-13.301
σ 7 10	69.956	- 24.100	-14.717
σ 7 11	4.026	0.293	0.584
σ 8 8	24.478	- 46.690	-33.116
σ 8 9	29.370	- 19.635	-10.877
σ 8 10	15.817	- 3.291	2.057
σ 8 11	- 3.386	4.880	3.243
σ 9 9	-13.918	- 3.383	- 4.978
σ 9 10	34.234	- 20.327	-14.643
σ 9 11	2.727	- 0.672	- 0.140
σ 10 10	39.701	-131.719	-155.167
σ 10 11	9.759	- 8.311	- 7.191
σ 11 11	0.405	- 0.995	- 0.724

Discussion

Again the null hypotheses of Leontief preferences and a homothetic (generalized Leontief) utility function are rejected.

It is interesting to note that once we are given the (average) number of hours a member of the labour force chooses to work in a given year, our model will endogenously determine the (aggregate) labour force participation rate.

Consumption-Labour Supply Model

Hypotheses:

(i) H_o:

$b_{oj} = 0 \; \forall_j$ (i.e., homothetic utility function)

$F(11, 181) = 16718.75$

Thus, reject H_o.

(ii) H_o:

$b_{ij} = 0 \; \forall_i \neq j$ (i.e., Leontief preferences)

$F(38, 181) = 520.857$

Thus, reject H_o.

CHAPTER VII

CONCLUSIONS

This study, at the very least, has contributed some interesting data to the area of Canadian consumer economics. On a slightly higher level, the empirical results, ranging from excellent fits in predicted expenditure shares* to Hicks-Allen elasticities of substitution, are important in that they provide a test of the revealed preference approach to the estimation of the underlying utility function.

A good result is that the predicted shares become more accurate as we progress further in time, as thus it is likely that the model may be useful for short-term projections of consumer demand.

The basic difficulty, as mentioned previously, is the aggregation problem. Although we may be able to prove the existence of individual utility functions and carry out analysis at the micro level, we have no guarantee that when we aggregate over individuals (or, for that matter, over goods) the aggregate utility function will exist. We ought, therefore, to recognize the existence of certain biases.

There is also the possibility that certain poor results (e.g., elasticities of substitution fluctuating more than would be expected) can be traced to weaknesses in the original data. Since some of our series are based on arbitrary assumptions (for example, the 'stock' of residential land)

---

*The only exception being motor vehicles in 1946, where the share is predicted to be negative. The actual share was very small, and thus it is easy enough to see that the model has not predicted well at the beginning.

there is the ever-present possibility that data anomalies have biased the estimated coefficients or at least distorted the overall results; nevertheless, we feel that these series are necessary and that, hopefully, certain of them can be improved upon. The Generalized Leontief Function appears to provide an excellent fit to the derived demand equations, and we hope to make use of this functional form in testing an extended model of consumer demand-labour supply behaviour. In fact, the next area to be explored will include human capital (i.e., the demand for higher education in an intertemporal *choice* framework) as well as physical capital (durables) and a more disaggregated treatment of the labour supply decision, hopefully extending to a group of occupations. Further, since one of the serious omissions of this study is the endogenous determination of savings, we hope to include such an element in a future model, perhaps accounting for the demand for financial assets and real balances. Perhaps, since we wish to avoid collinearity, we will aggregate the nondurables into a single category in future work. It should be noted that elasticities of demand can be calculated directly from the σ_{ii} and estimated shares.

It is perhaps of interest to examine another approach to "leisure." Nordhaus and Tobin [28], in developing a new measure of economic welfare, attempt to quantify some of the important differences between goods and services produced in the marketplace (i.e., what GNP measures) and those actually available for ultimate consumer satisfaction (i.e., consumption). The most important factor in the difference between production and welfare turns out to be the value of leisure time:

> The omission of leisure and of nonmarket productive activity from measures of production conveys the impression that economists are blindly materialistic. Economic theory teaches that welfare could rise, even while NNP falls, as the result of voluntary choices to work for pay fewer hours per week, weeks per years, years per lifetime.[1]

The wage rate, as in the present study, is chosen to be the price for leisure; however, they go one step beyond in making imputations for nonmarket activity as well. In the end, using their imputations for the time elements in consumption, the authors calculate a measure of economic welfare (MEW), several variants of which are compared with NNP as to growth over time. The basic effect of their imputations for leisure is a reduction in the growth rate in spite of increasing leisure consumption.

In closing, we will examine the indirect utility function obtained by evaluating the inverse indirect utility function h(v) and inverting it. This is evaluated at the prevailing prices (v_i) in each year and with the estimated b_{ij}.

---

[1] Nordhaus and Tobin [28; p. 9].

TABLE VII-1

INDIRECT UTILITY FUNCTION

g(Y; p)

YEAR	CONSUMPTION ONLY	CONSUMPTION AND LEISURE
1946	2.217888	6.194567
1947	2.223509	6.189904
1948	2.222746	6.188863
1949	2.224744	6.185479
1950	2.233116	6.181176
1951	2.237154	6.180198
1952	2.246631	6.177883
1953	2.257169	6.176811
1954	2.263582	6.176455
1955	2.276941	6.176762
1956	2.293592	6.177373
1957	2.301910	6.178276
1958	2.310517	6.180124
1959	2.323212	6.181379
1960	2.332206	6.182951
1961	2.339212	6.184282
1962	2.345377	6.185060
1963	2.356039	6.186886
1964	2.370543	6.188728
1965	2.383481	6.190127
1966	2.392507	6.190932
1967	2.399877	6.192188
1968	2.407138	6.195193
1969	2.417972	6.197188

In the consumption model, the standard of living (a proxy for real per-capita consumption), except for a slight dip in 1948, is rising over time. When leisure is included, the value of the indirect utility function declines slightly until 1954 and then steadily rises. This is not unbelievable, since the early 1950's were associated with little economic growth.

A test of the accuracy of these indirect utility functions is that the 1961 values, following our normalization (III-3) ought to be $\frac{1}{k^2}$ or $\frac{1}{V^*}$. Clearly, the following comparison indicates that our model has performed well:

CONSUMPTION ONLY	CONSUMPTION AND LEISURE
$V^*_{1961} = .4275$	$V^*_{1961} = .1617$
$\frac{1}{V^*} = 2.33918$	$\frac{1}{V^*} = 6.18429$
$g(Y;p)_{1961} = 2.33921$	$g(Y;p)_{1961} = 6.18428$

BIBLIOGRAPHY

A. REFERENCES

(1) Allen, R. G. D. *Mathematical Analysis for Economists.* London: MacMillan and Co. Ltd., 1937.

(2) Becker, Gary S. "A Theory of the Allocation of Time," *Economic Journal*, 75 (1965), pp. 493-517.

(3) Bowles, Samuel, David Kendrick, Lance Taylor and Marc Roberts. *Notes and Problems in Microeconomic Theory.* Chicago: Markham Publishing Co., 1970.

(4) Bridge, J. L. *Applied Econometrics.* Amsterdam: North Holland Publishing Co., 1971.

(5) Christensen, Laurits, and Dale Jorgenson. "The Measurement of U.S. Real Capital Input, 1929-1967," *Review of Income and Wealth* (December, 1969).

(6) Debreu, Gerard. *Theory of Value.* New York: Wiley, 1959.

(7) Diewert, W.E. "An Application of the Shephard Duality Theorem: A Generalized Leontief Production Function," *Journal of Political Economy,* 79 (May-June, 1971), pp. 481-507.

(8) ____________. "Canadian Labour Markets: A Neoclassical Econometric Approach," Research Branch, Program Development Service, Department of Manpower and Immigration, Ottawa, 1969.

(9) ____________. "Choice on Labour Markets and the Theory of the Allocation of Time." Research Branch, Program Development Service, Department of Manpower and Immigration, Ottawa, 1971.

(10) ____________. "Intemporal Consumer Theory and the Demand for Durables," Discussion Paper No. 53, Department of Economics, University of British Columbia, April, 1971.

(11) ____________. "Walras' Theory of Capital Formation and the Existence of a Temporary Equilibrium," Research Branch, Program Development Service, Department of Manpower and Immigration, Ottawa, 1972.

(12) ____________. Unpublished Lecture Notes -- Mathematical Economics 515, University of British Columbia, Winter, 1970.

[13] Evans, Michael K. *Macroeconomic Activity: Theory, Forecasting Control*. New York: Harper and Row, 1969.

[14] Firestone, O. J. *Canada's Economic Development*. London: Bowes and Bowes, 1958.

[15] ____________. *Residential Real Estate in Canada*. Toronto: University of Toronto Press, 1951.

[16] Friedman, Milton. *A Theory of The Consumption Function*. Princeton, N.J.: Princeton University Press, 1957.

[17] Friend, Irwin, and Robert Jones (Eds.). *Study of Consumer Expenditures, Incomes, and Savings*. Wharton School of Finance, University of Pennsylvania, 1960.

[18] Goldberger, Arthur S. "Functional Form and Utility: A Review of Consumer Demand Theory," *Systems Formulation, Methodology, and Policy Workshop Paper 6703*, Social Systems Research Institute, University of Wisconsin, 1967.

[19] Goldsmith, Raymond W. *The National Wealth of The United States in The Postwar Period*. National Bureau of Economic Research, New York, 1962.

[20] Gussman, T. K. "On Occupational Wages: A Converter for Intercensial Years." Working Paper #13, Projections Studies Unit, Research Branch, Program Development Service, Department of Manpower and Immigration, Ottawa, 1971.

[21] Hamburger, M. "Interest Rates and the Demand for Consumer Durable Goods," *American Economic Review*, 57 (December, 1967), pp. 1131-1153.

[21aa] Hanoch, Giora. "Generation of New Production Functions Through Duality," mimeo, 1970.

[21a] Harberger, Arnold C. (Ed.). *The Demand for Durable Goods*. Chicago: University of Chicago Press, 1960.

[22] Helliwell, J. F. *et al.* "Quarterly Estimates of Private Sector Wealth," Research Discussion Paper No. 18, Reserve Bank of Australia, 1971.

[23] ____________., H. T. Shapiro, G. R. Sparks, I. A. Stewart, F. W. Gorbet and D. R. Stephenson. *The Structure of RDX2*. Ottawa, Bank of Canada, 1971.

[24] Hicks, J. R. *Value and Capital*. Oxford: At the Clarendon Press, 1946.

[25] Houthakker, H., and Lester Taylor. *Consumer Demand in The United States*. (Second edition). Cambridge, Mass.: Harvard University Press, 1970.

[26] Johnston, J. *Econometric Methods*. New York: McGraw-Hill, 1963.

[27] Lancaster, Kelvin. *Mathematical Economics*. New York: MacMillan, 1968.

[28] Nordhaus, William D., and James Tobin. "Is Growth Obsolete?" in *Economic Growth*, National Bureau of Economic Research (Fiftieth Anniversary Colloquium V), New York, 1972.

[29] Pickett, James. "Residential Capital Formation in Canada 1871-1921," *Canadian Journal of Economics and Political Science*, Vol. 29, No. 1 (February, 1963), pp. 40-58.

[30] Poapst, J. V., and W. R. Waters. "Individual Investment: Canadian Experience," *Journal of Finance*, Vol. XVIII, No. 4 (December, 1963), pp. 647-665.

[31] ______________________________ . "Rates of Return on Consumer Durables," *Journal of Finance*, Vol. XIX, No. 4 (December, 1964).

[32] Pollak, Robert. "The Demand for Consumer Durables," Department of Economics Seminar, University of British Columbia, March 14, 1972.

[33] Schweitzer, Thomas T. *Personal Consumer Expenditures in Canada, 1926-75* (Parts I and II). Staff Study No. 26, Economic Council of Canada. Ottawa: Queen's Printer, 1970.

[33a] Shephard, Ronald W. *Cost and Production Functions*. Princeton, N.J.: Princeton University Press, 1953.

[33b] _________________ . *Theory of Cost and Production Functions*. Princeton, N.J.: Princeton University Press, 1970.

[34] Steele, Marion. "Determinants of Owner-Occupied Dwelling Starts: Canada, 1921-1940." Department of Economics, University of Guelph, 1972.

[35] Stone, Richard and D. A. Rowe. "The Market Demand for Durable Goods," *Econometrica*, Vol. XXV, No. 3 (1957), pp. 423-443.

[36] Usher, Dan. "Measuring Real Consumption From Quantity Data, Canada 1935-1968. (Mimeo). To be included as a chapter in a forthcoming book.

[37] Wales, T. J. "A Generalized Linear Expenditure Model of The Demand for Nondurable Goods in Canada," *Canadian Journal of Economics* (November, 1971), pp. 1471-1484.

[38] Weber, Warren E. "The Effect of Interest Rates on Aggregate Consumption," *American Economic Review*, 55 (September, 1970), pp. 591-600.

[39] ______________ . "Interest Rates, Relative Prices, and Consumer Expenditure for Durables and Nondurables: A Multi-period Utility Maximization Approach." Department of Economics, Virginia Polytechnic Institute and State University, 1972.

[40] Wilson, G. W., Scott Gordon, and Stanislaw Judek. *Canada: An Appraisal of Its Needs and Resources.* Twentieth Century Fund, New York, 1965.

[41] Zellner, A. "An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests For Aggregation Bias," *Journal of The American Statistical Association*, Vol. LVII (1962), pp. 348-368.

B. DATA SOURCES

Canada, Statistics Canada (formerly the Dominion Bureau of Statistics), Ottawa: The Queen's Printer.

	Catalogue Number	Title
[1]	11-003	*Canadian Statistical Review* (monthly)
[2]	11-206	Supplement to *Canadian Statistical Review*
[3]	13-502	National Accounts, Income and Expenditure, 1926-1956
		National Income and Expenditure Accounts 1926-1968 (revised system), August, 1969
[4]	62-002	Prices and Price Indices
[5]	62-517	City Family Expenditure, 1957
[6]	62-519	Residential Rents in Major Canadian Cities, September, 1959. Published April, 1962
[7]	62-521	Urban Family Expenditure, 1959
[8]	62-525	Urban Family Expenditure, 1962
[9]	62-527	Urban Family Expenditure, 1964
[10]	64-002	New Residential Construction
[11]	64-202	Household Facilities and Equipment

	Catalogue Number	Title
[12]	91-202	Estimated Population by Age and Sex Groups, Canada and the Provinces
[13]	91-511	Population by Sex and Age

Uncatalogued Data

[14]	National Accounts -- Income and Expenditure	
	Table 44:	Direct Taxes - Persons, 1926-1969
	Table 46:	Indirect Taxes - 1926-1969
	Table 53:	Personal Expenditures on Consumer Goods and Services in Current Dollars - 1926-1969
	Table 54:	Personal Expenditure on Consumer Goods and Services in Constant (1961) Dollars, 1926-1969
[15]	Housing Capital Stock, 1921-1971, Housing and Building Permits Section, Business Finance Division, Statistics Canada, May, 1972	

Other Sources

[16] Bank of Canada, *Statistical Summary* (monthly).

[17] *Canadian Consumer Credit Factbook.* Canadian Consumer Loan Association, Toronto, 1972.

[18] *Economic Review.* Ottawa, Ministry of Finance, April, 1972.

[19] *Housing in Canada.* Central Mortgage and Housing Corporation, Annual to 1965.

[20] *National Housing Statistics.* Central Mortgage and Housing Corporation, Annual from 1956.

[21] *The National Finances 1970-71.* Canadian Tax Foundation, Toronto, 1970.

[22] Urquhart, M. C. (Ed.), and K. A. H. Buckley (Asst. Ed.), *Historical Statistics of Canada.* Toronto: MacMillan, 1971.